Cosmology with Fuzzy Dark Matter Model

Xinyu Li 李昕宇
Canadian Institute for Theoretical Astrophysics
Perimeter Institute
with Lam Hui, Greg Bryan, Austin Joyce

CITA | ICAT
Canadian Institute for Theoretical Astrophysics
L'institut Canadien d'astrophysique theorique

PERIMETER INSTITUTE

IN LVMINE TVO VIDEBIMVS LVMEN

“Fuzzy” Dark Matter

- Cold dark matter is good on large scales (>10kpc), but have problems on small scales. e.g. the missing satellite problem and the core-cusp problem.

- FDM is alternative dark matter model composed of ultralight bosons/axions described by a classical coherent wave function with macroscopic de Broglie wavelength (~kpc).

- FDM acts just like CDM on scales much larger than then de Broglie wavelength, but will change the small scale structure.

Dynamics of FDM

- Schrodinger-Poisson equation

$$i\hbar\left(\partial_t\psi + \frac{3}{2}H\psi\right) = \left(-\frac{\hbar^2}{2ma^2}\nabla^2 + m\Phi\right)\psi$$

- Madelung (fluid) formalism

$$\dot{\rho} + 3H\rho + \frac{1}{a}\nabla\cdot(\rho v) = 0\,,$$

$$\dot{v} + Hv + \frac{1}{a}(v\cdot\nabla)v = -\frac{1}{a}\nabla\Phi - \frac{\hbar^2}{2m^2a^3}\nabla p\,,$$

$$\boxed{\psi \equiv \sqrt{\frac{\rho}{m}}e^{i\theta} \quad , \quad v \equiv \frac{\hbar}{ma}\nabla\theta\,. \qquad p \equiv -\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = -\frac{1}{2}\nabla^2\log\rho - \frac{1}{4}(\nabla\log\rho)^2\,.}$$

- De Broglie Length Scale

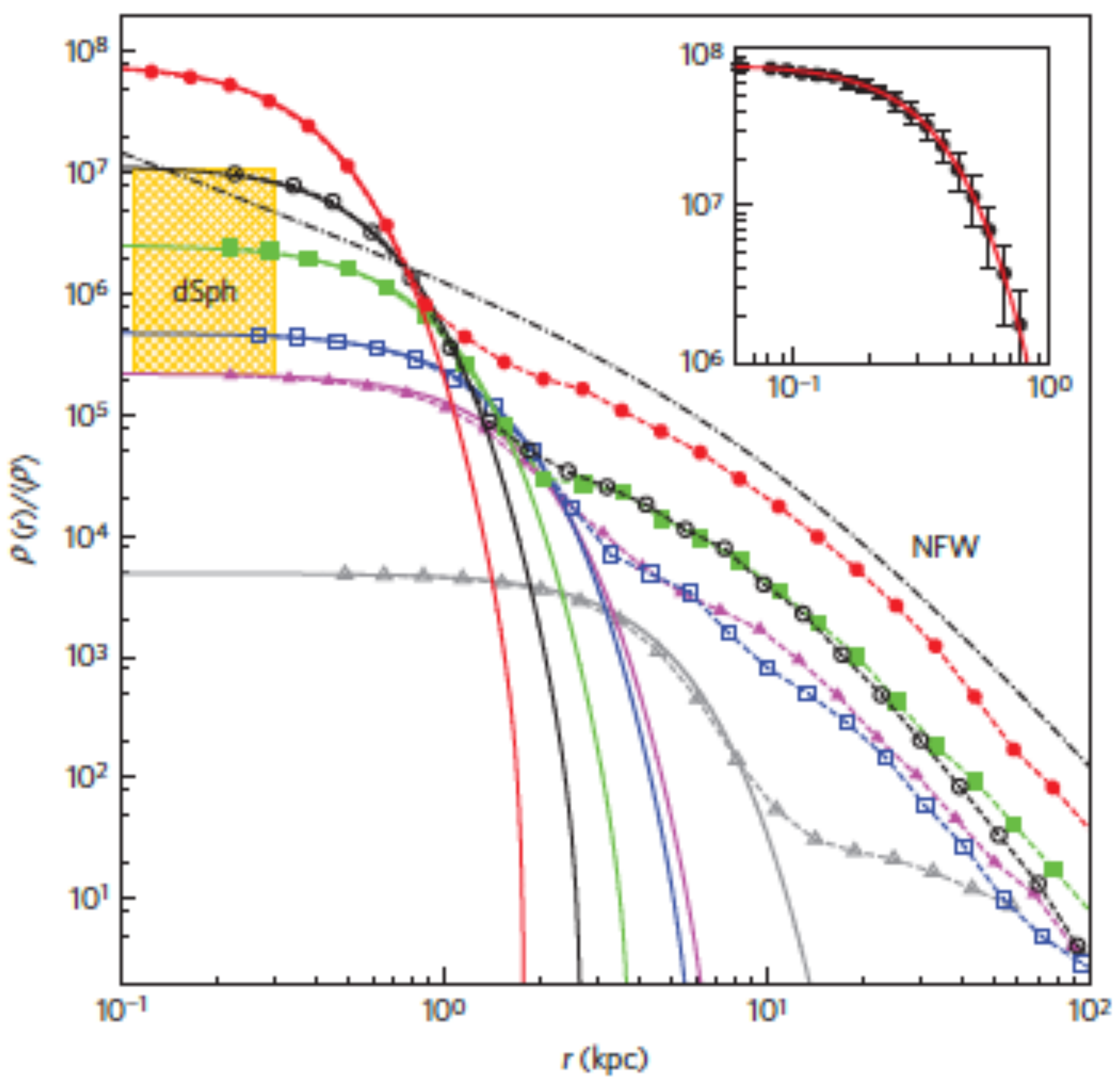
$$\frac{\lambda}{2\pi} = \frac{\hbar}{mv} = 1.92\ \mathrm{kpc}\left(\frac{10^{-22}\ \mathrm{eV}}{m}\right)\left(\frac{10\ \mathrm{km\,s^{-1}}}{v}\right)$$

- Jeans Length Scale

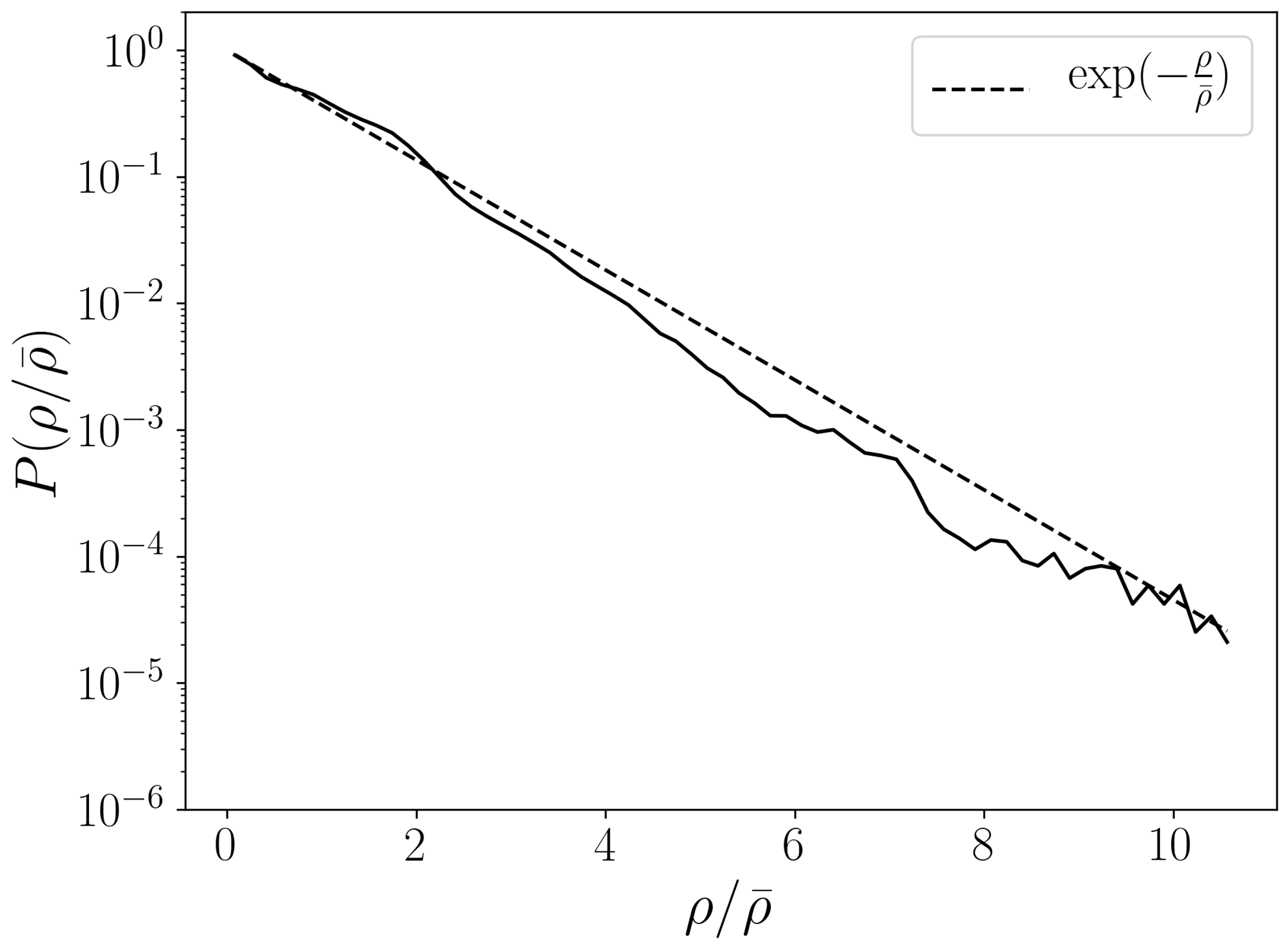
$$\begin{aligned} r_J &= 2\pi/k_J = \pi^{3/4}(G\rho)^{-1/4}m^{-1/2}, \\ &= 55 m_{22}^{-1/2}(\rho/\rho_b)^{-1/4}(\Omega_m h^2)^{-1/4}\ \mathrm{kpc}, \end{aligned}$$

FDM halo

A soliton core forms at the halo center.
Probability distribution of density follows the Gaussian model.

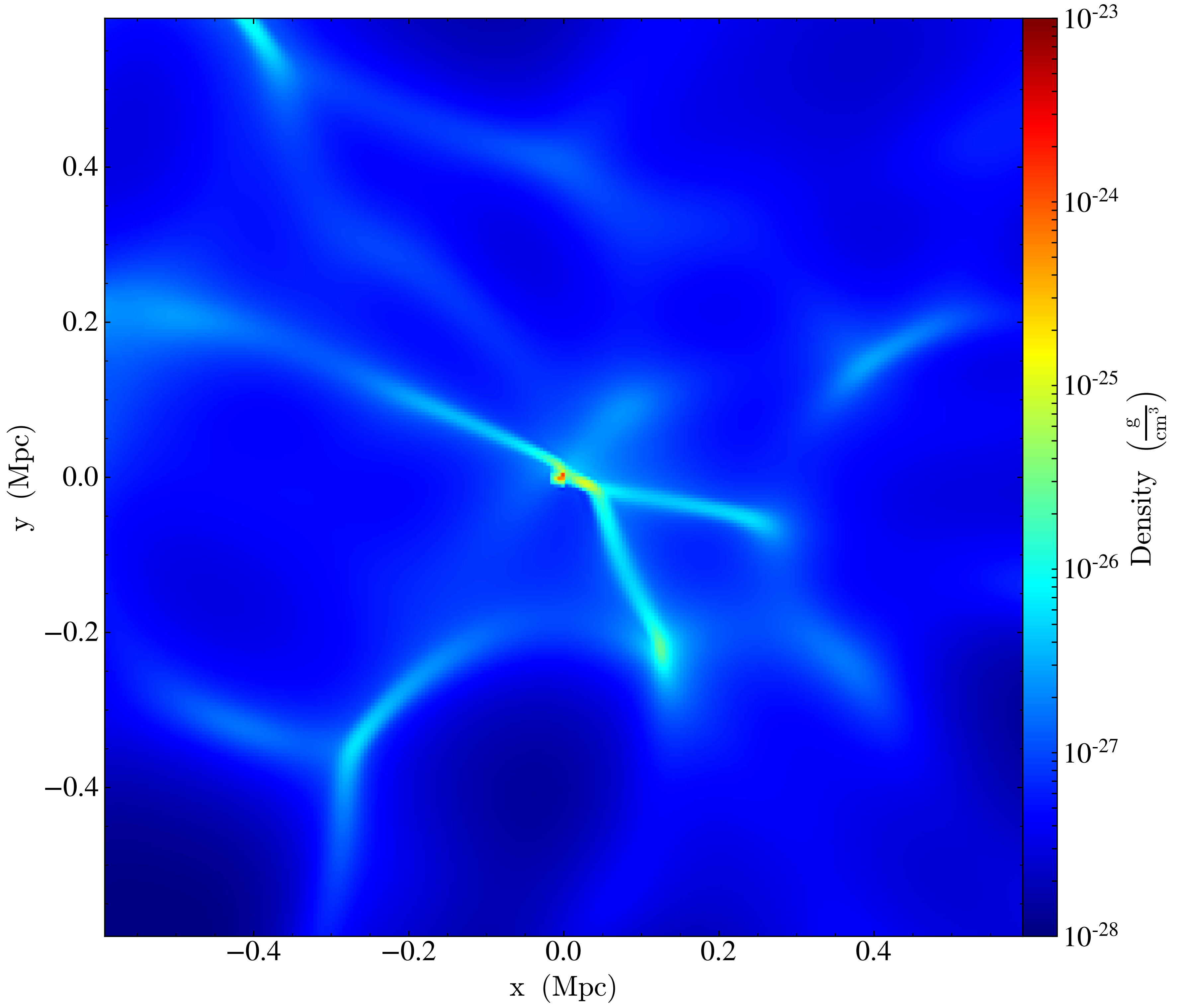


Schive et al. 2014

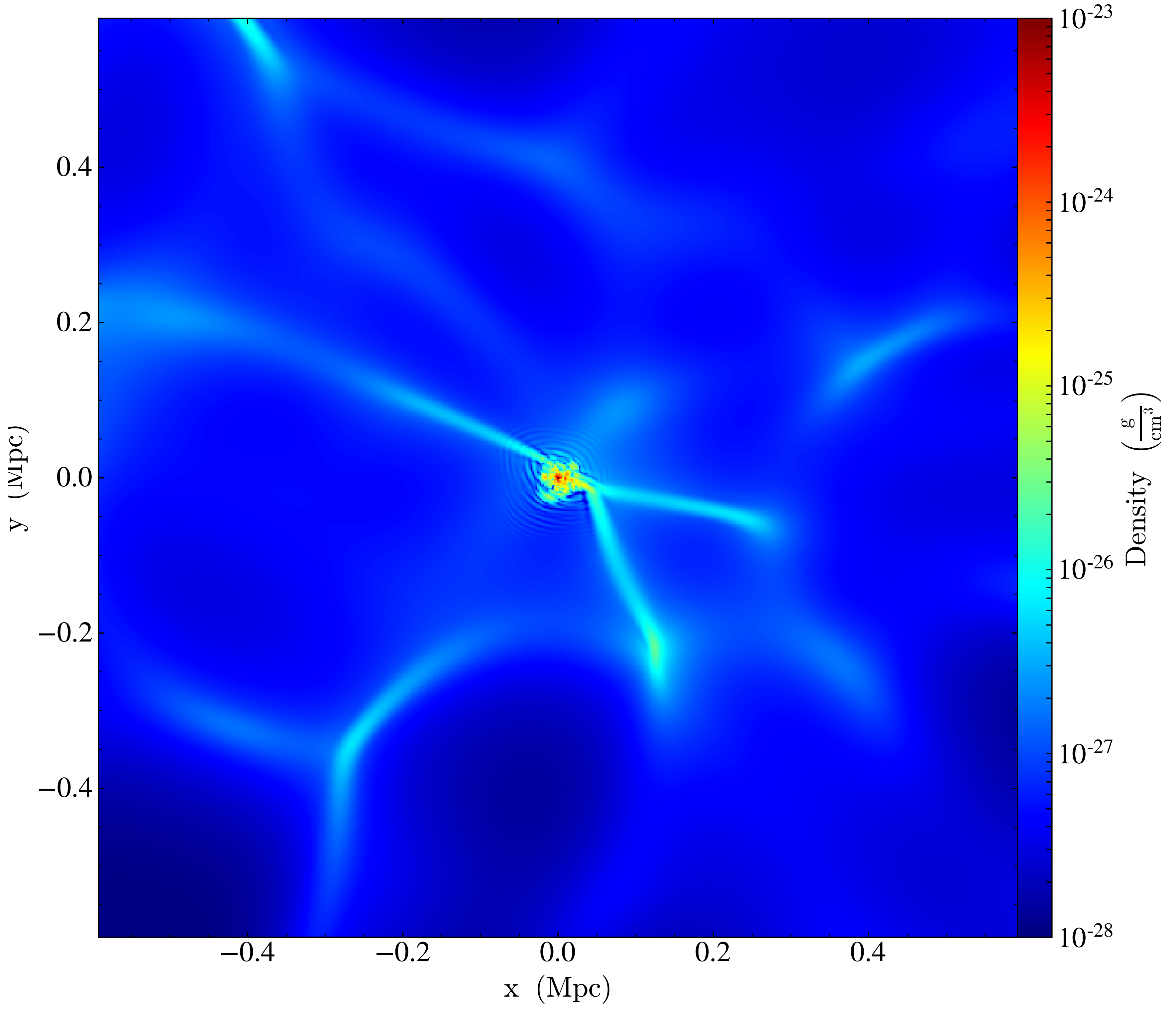


Fluid vs Wave Simulations

SPoS code: 1810.01915

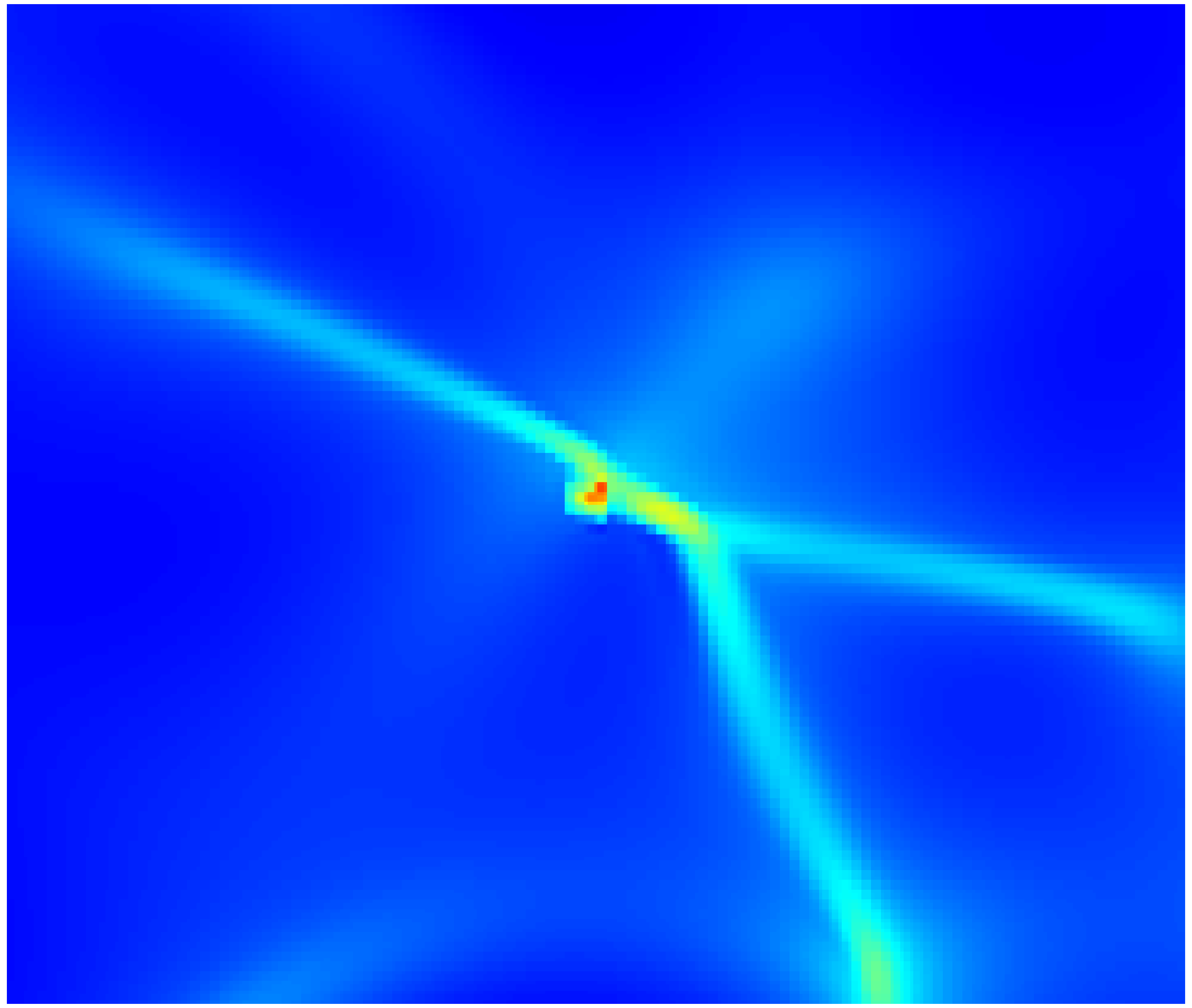


Fluid

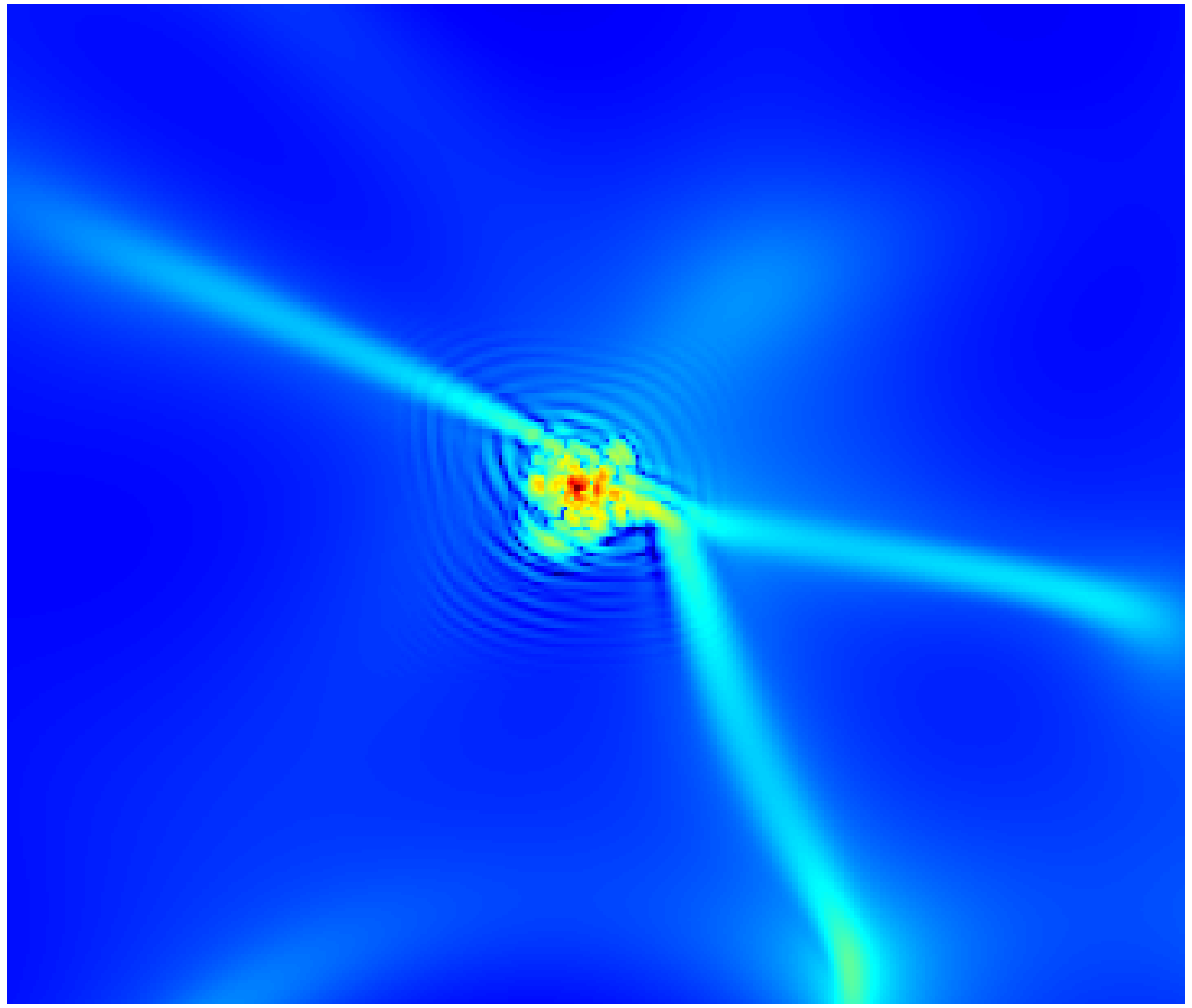


Wave

Fluid vs Wave Simulations



Fluid



Wave

The wave nature of FDM leads to interesting new phenomena!

Vortex lines (arXiv:2004:01188)

- Let us examine the Madelung representation again

$$\psi \equiv \sqrt{\frac{\rho}{m}} e^{i\theta} \quad , \quad v \equiv \frac{\hbar}{ma} \nabla \theta \, .$$

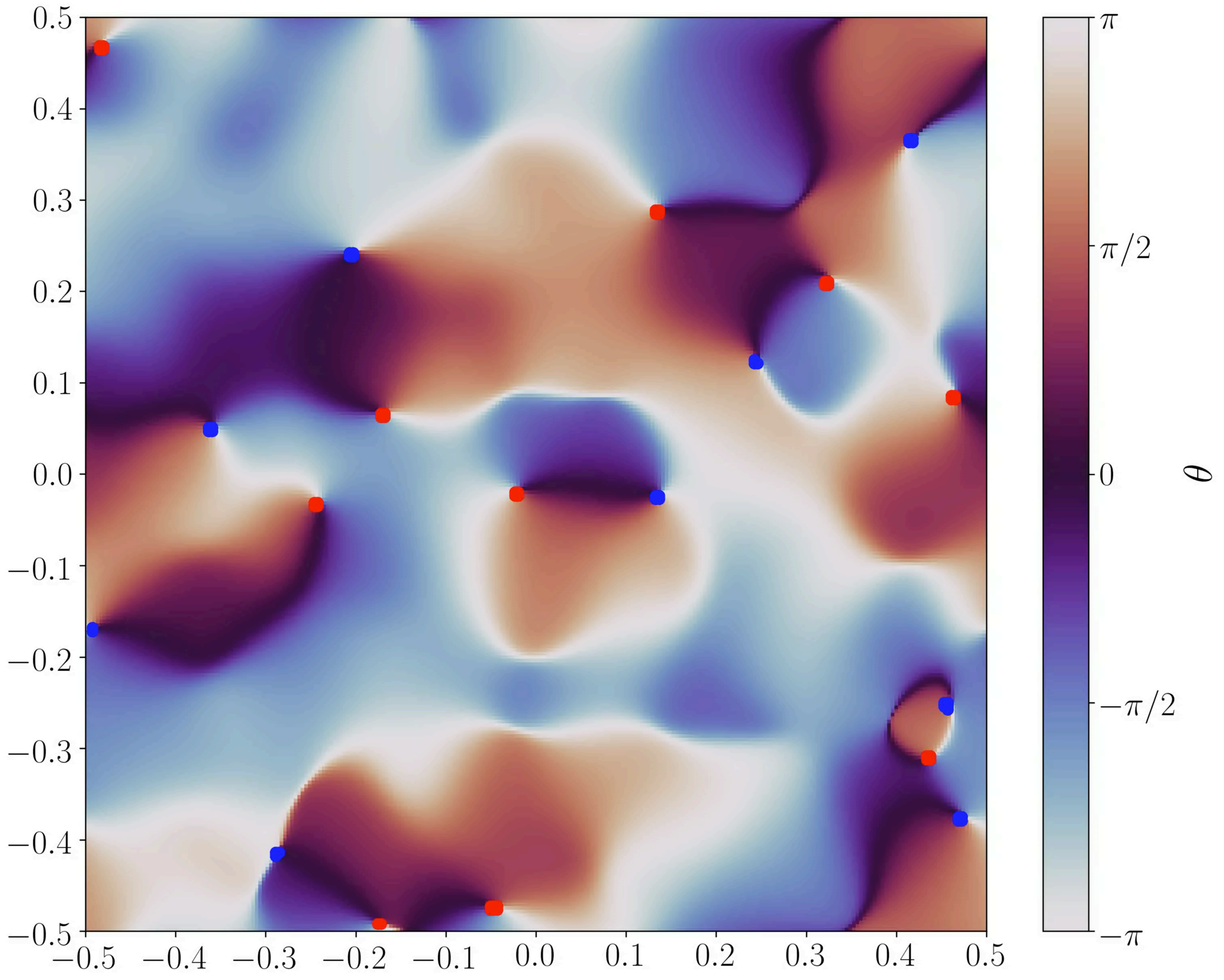
- The phase is not well defined when $\Psi = 0$ => topological defects.

- $\Psi = 0$ requires both the real and imaginary parts to vanish. In 3D, they occur at the intersection of two surfaces ($Re\Psi = 0$ and $Im\Psi = 0$) $--$ 1D structure.

Numerical Realizations — 2D

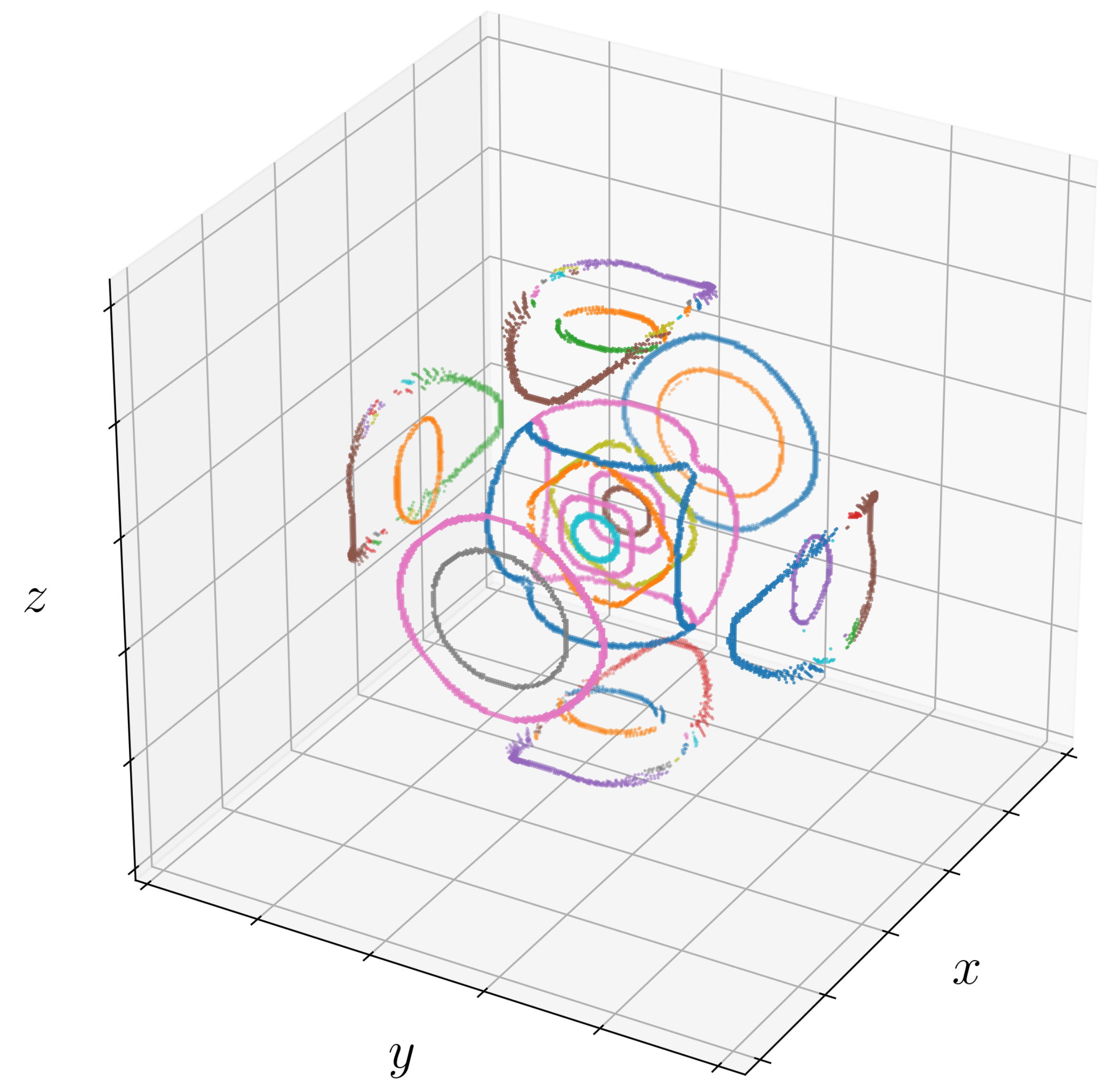
- Initial condition: Real and Imaginary are independent Gaussian with spectrum e^{-k^2/k_{max}^2}

- We follow the dynamics of the Schrodinger equation.

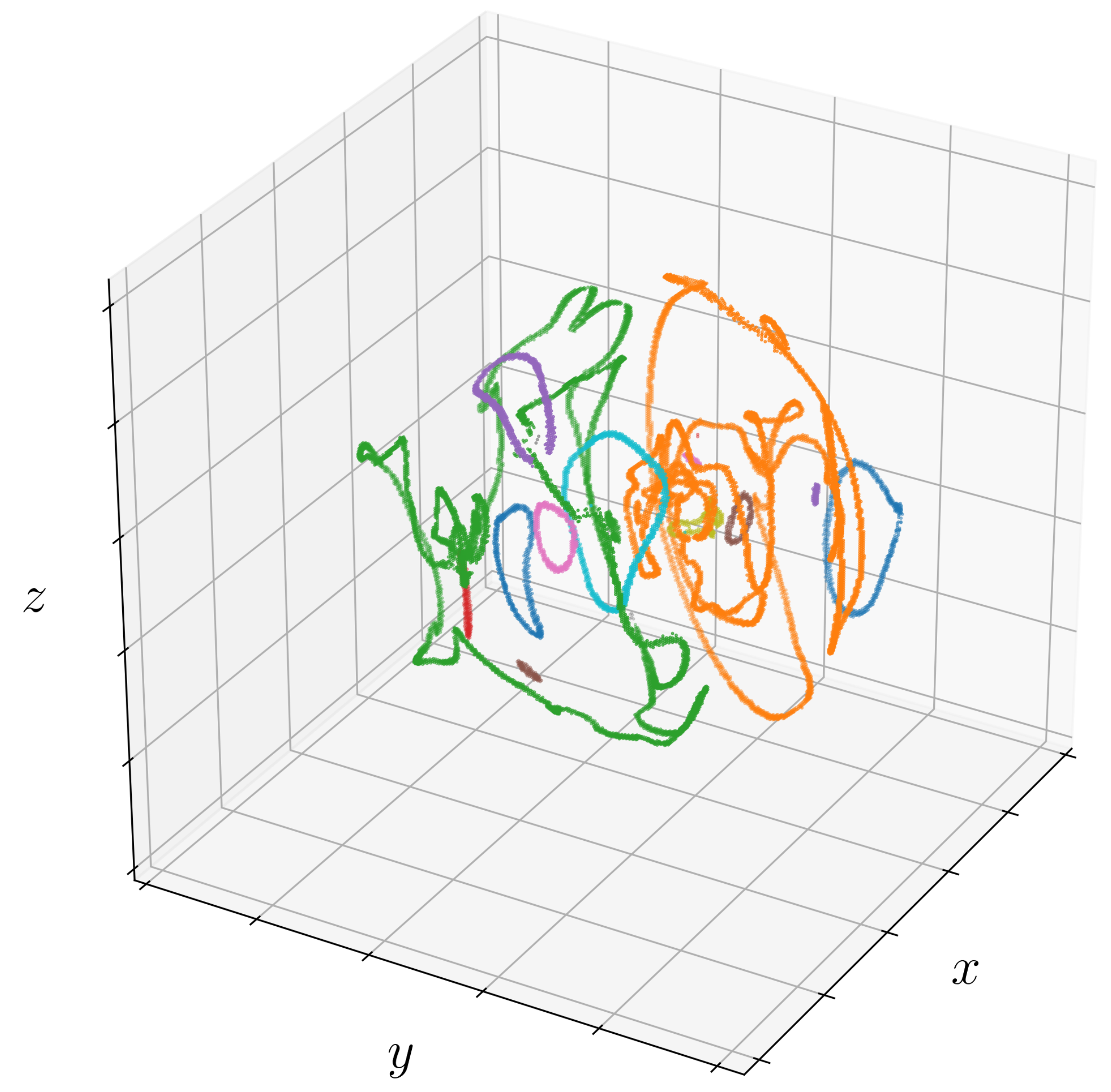


Numerical Simulations with Gravity

- Vortex lines emerge from initial condition with no angular momentum.
- The typical size of vortices is found to be the de Broglie wavelength.
- Expect to have one vortex line per de Broglie wavelength.



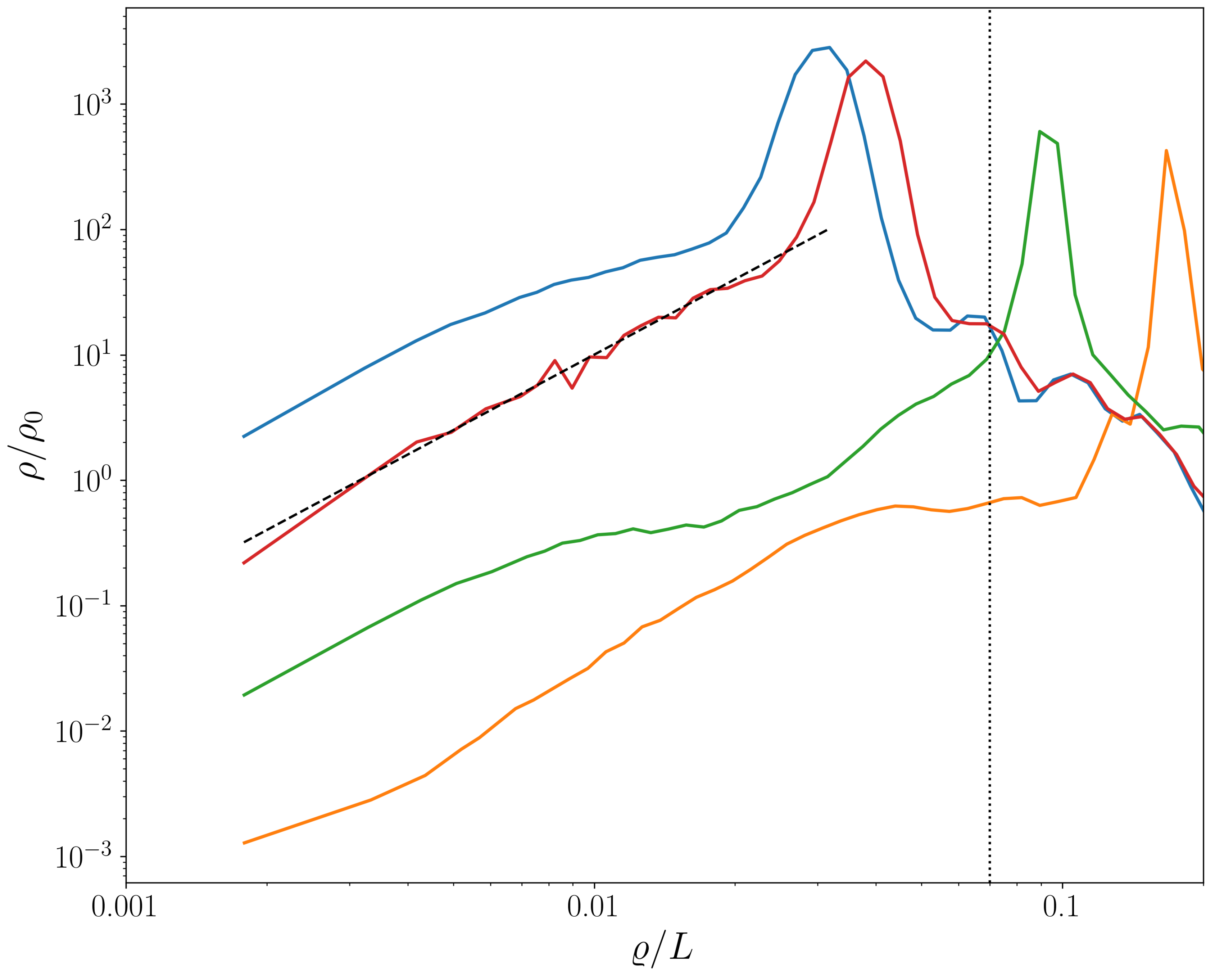
Symmetric initial condition



Random initial condition

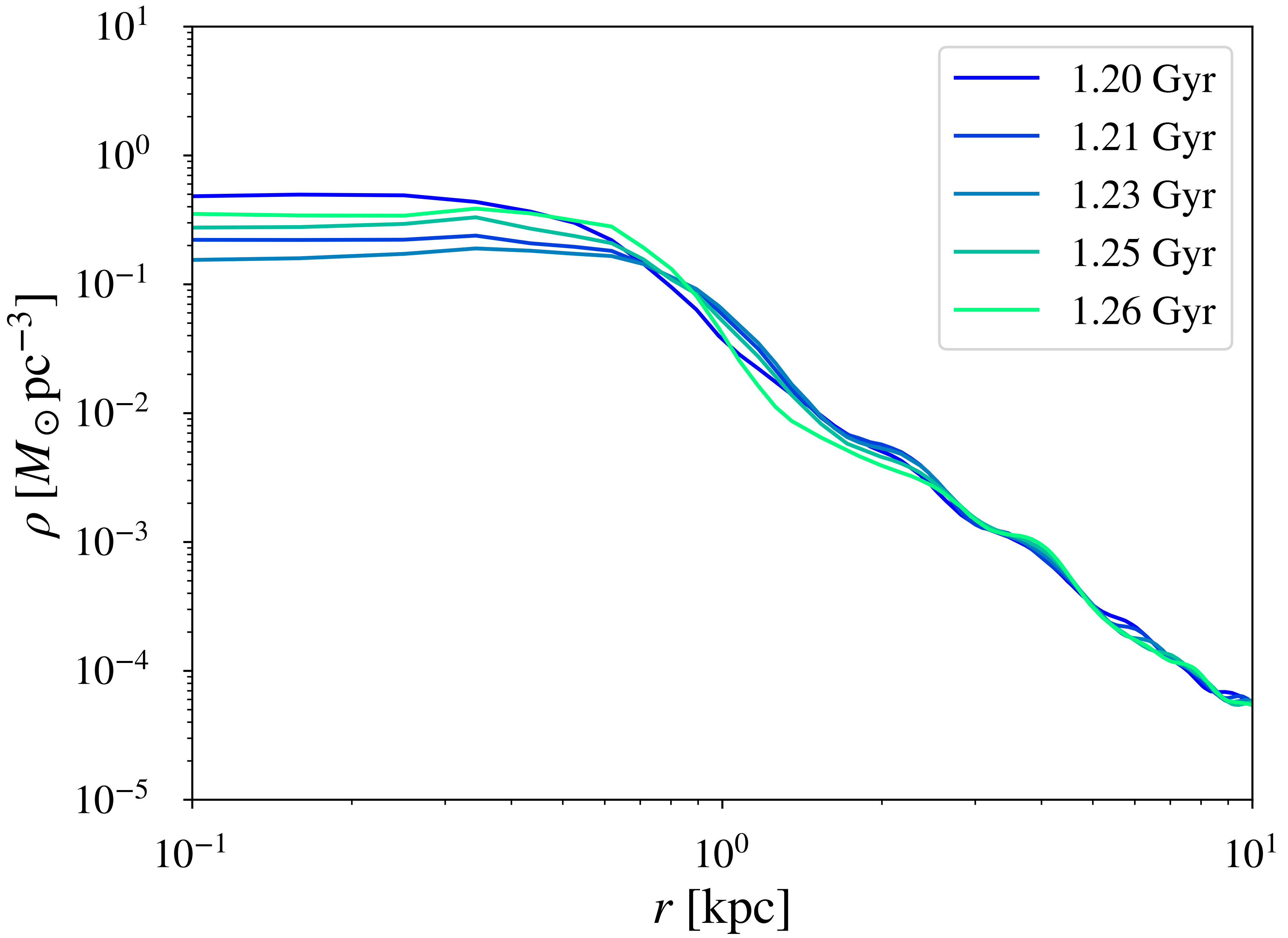
Density profiles of vortex lines

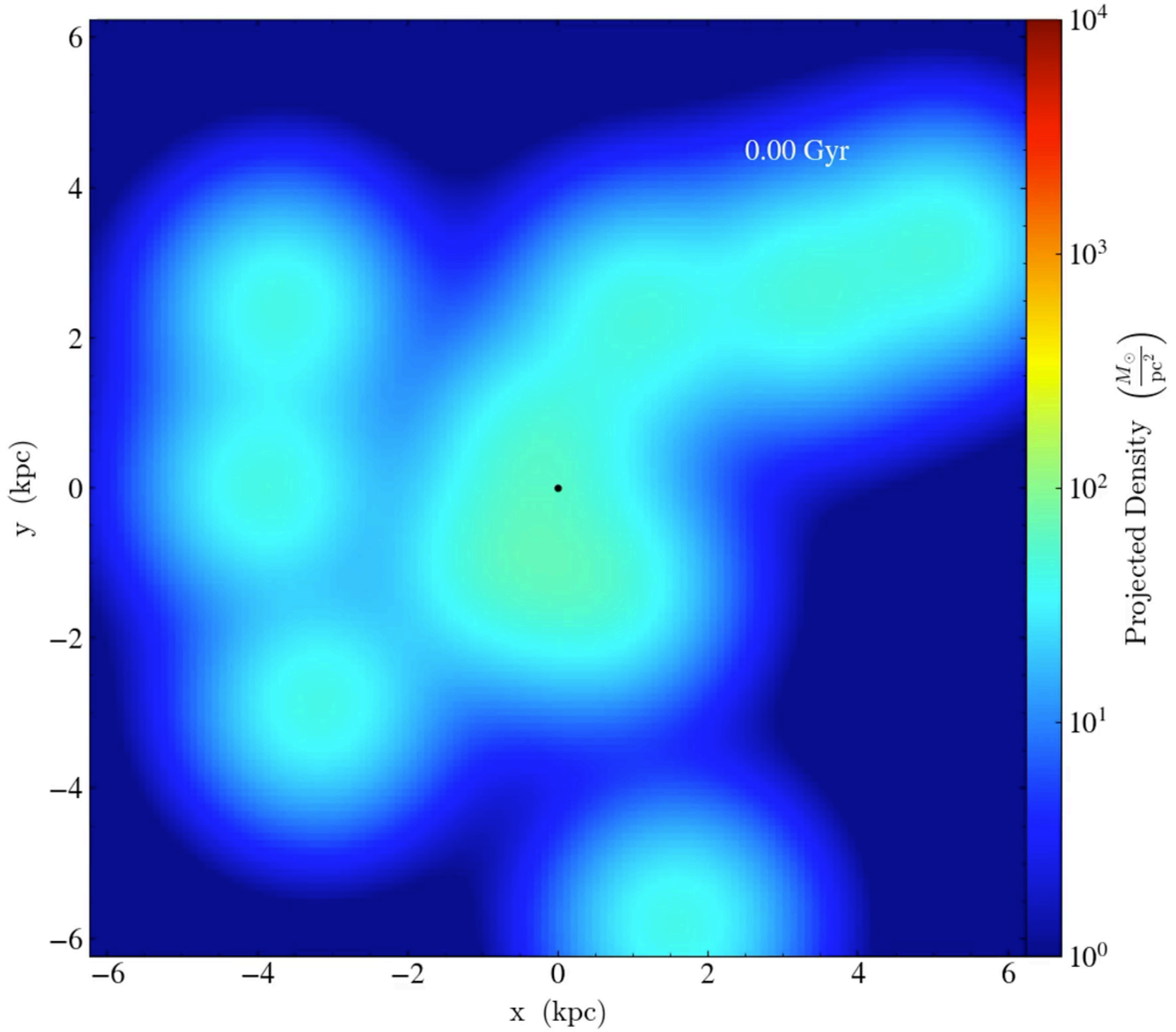
Density increases as r^2 from the zero density centre.



Soliton oscillation and random walk

The soliton is observed to oscillate and random walk around the halo centre.

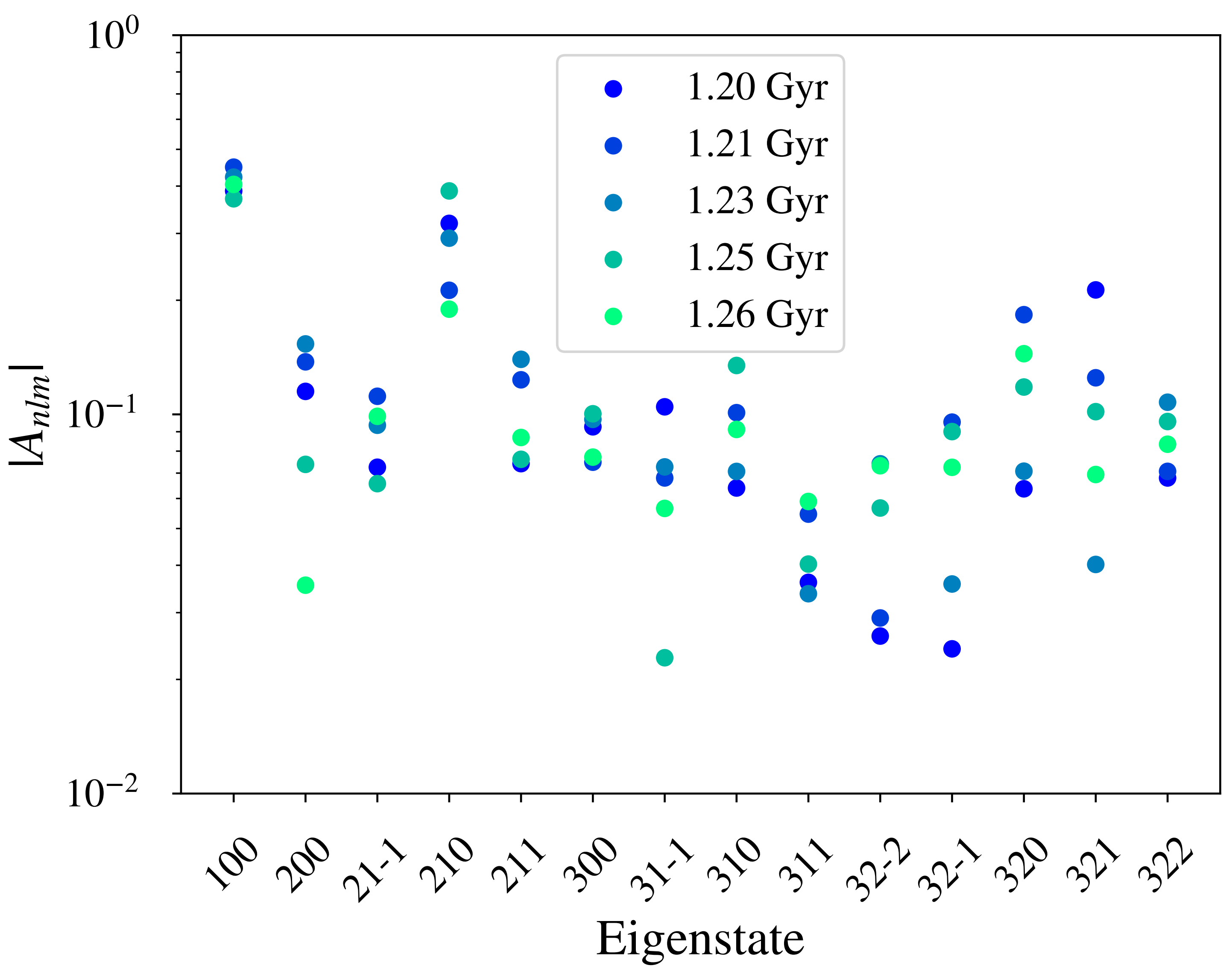




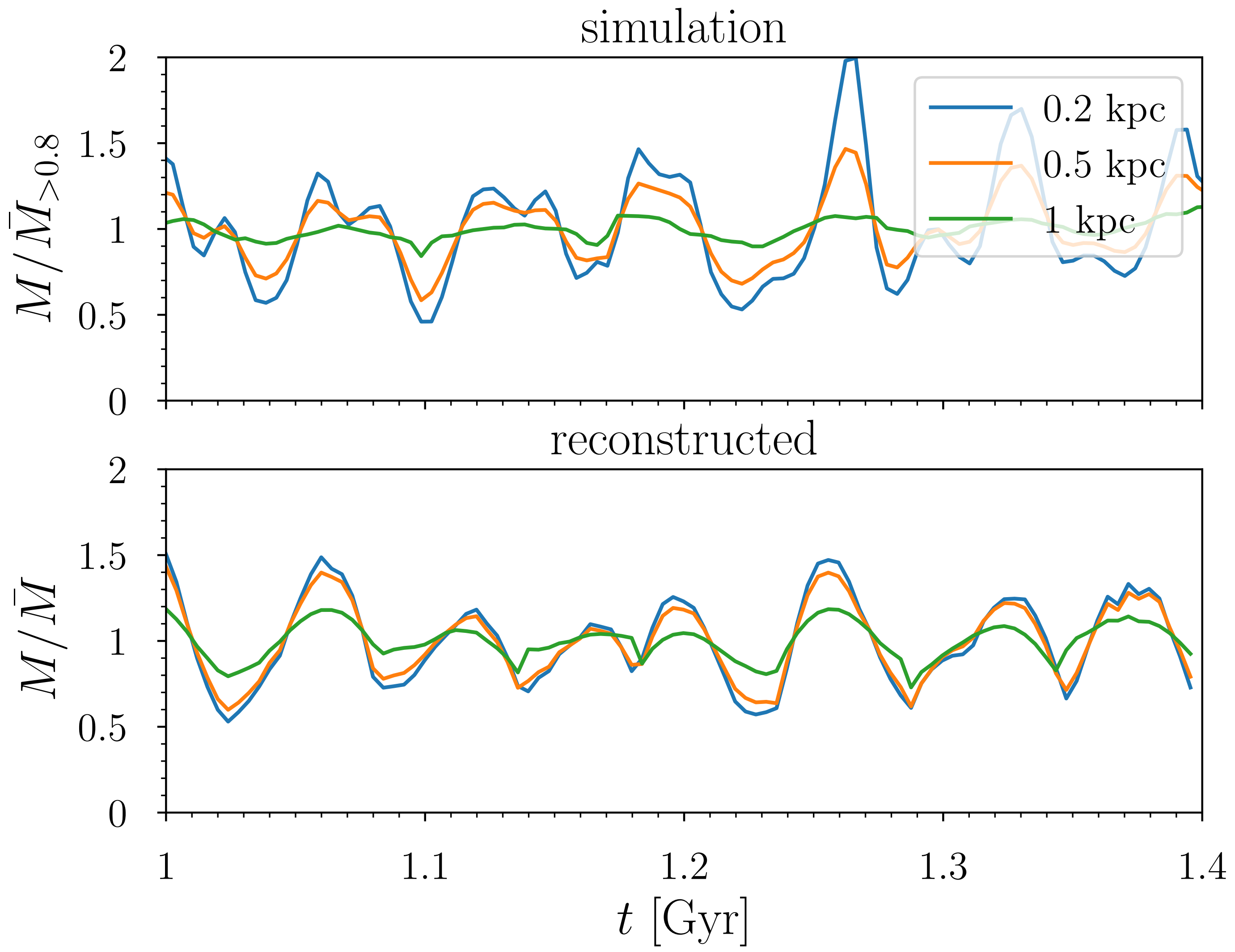
Soliton oscillation and random walk

The origin of both phenomena is still the interference between eigenstates.

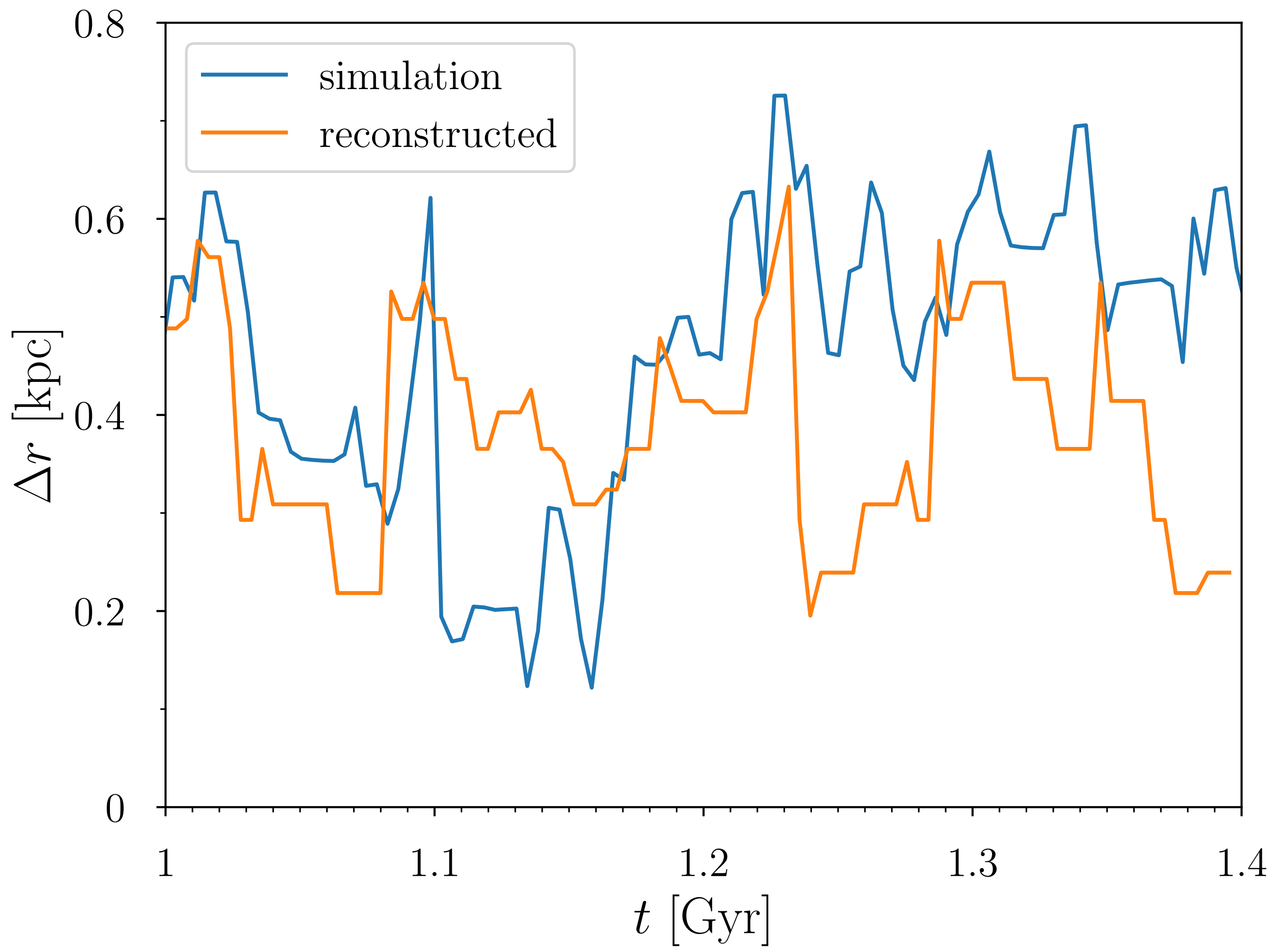
The FDM halo maintains a static gravitational potential, but its wave function will **not** relax to the ground state during the collapse.



Eigenstates decomposition



Soliton oscillation



Random Walk

Observational Signature

- Vortex lines: Micro(de)lensing, wave lensing, flux anomaly, variation of pulsar timing, Shapiro delay of pulses

$$\Delta\mu_{\text{pert.}}^{-1} \lesssim \sqrt{\frac{\lambda_{\rm c}}{R_{\rm halo}}} \sim \begin{cases} 0.007\left(\frac{10^{-22}\,\text{eV}}{m}\right)^{1/2}\left(\frac{1000\,\text{km/s}}{v}\right)^{1/2}\left(\frac{1\,\text{Mpc}}{R_{\rm halo}}\right)^{1/2} & \text{cluster}\,, \\ 0.06\left(\frac{10^{-22}\,\text{eV}}{m}\right)^{1/2}\left(\frac{250\,\text{km/s}}{v}\right)^{1/2}\left(\frac{50\,\text{kpc}}{R_{\rm halo}}\right)^{1/2} & \text{galaxy}\,, \end{cases}$$

- Soliton oscillation: dynamical effects, heating of stellar streams and clusters.

- GD-1 stream: 1km/s velocity perturbation, density power spectrum. Possible with future Gaia and LSST data.

Method	Constraint	Sources of systematic uncertainties	Refs.
Lyman-alpha forest	$m > 3 \times 10^{-21}$ eV	Ionizing background/temp. fluctuations	1
Density profile	$m > 10^{-21}$ eV	Baryonic feedback/black hole	2
Satellite mass	$m > 6 \times 10^{-22}$ eV	Tidal stripping	3
Satellite abundance	$m > 2.9 \times 10^{-21}$ eV	Subhalo mass function prediction	4

References: 1=Iršič et al. (2017), Kobayashi et al. (2017), Armengaud et al. (2017), 2=Bar et al. (2018), 3=Safarzadeh & Spergel (2019), 4=Nadler et al. (2020). See text on the methodology and systematic uncertainties of each constraint.

Conclusion

- The FDM model is promising in solving the small scale problem in the CDM model.
- The wave nature of FDM predicts new phenomena that can be tested observationally.
- Many interesting problems remain to be worked out!

Thank you for your attention!

- Kamionkowski and Liddle (2000): a sharp cut-off at 4.5h/Mpc can solve the over-abundance of low mass halos.

- Linear power spectrum of FDM (Hu, Barkana & Gruzinov 2000)

$$P_{\rm FCDM}(k) = T_{\rm F}^2(k) P_{\rm CDM}(k), \qquad T_{\rm F}(k) \approx \frac{\cos x^3}{1 + x^8}$$

- A cut-off at k~4.5Mpc^{-1} re

$$k_{1/2} \approx \frac{1}{2} k_{J\rm eq} m_{22}^{-1/18} = 4.5 m_{22}^{4/9}\ \mathrm{Mpc}^{-1}$$

Outline

1. Numerical simulations of FDM

- comparison between wave and fluid formulation
- application to Lyman-α flux spectrum

2. Vortex line solutions

- analytical and numerical solutions
- possible observational signatures

3. Future Work

Existing simulations

- Wave formulation: Schive et al., Mocz et al., Schwabe et al.
- Fluid formulation (SPH): Zhang et al., Veltmaat et al., Nori & Baldi
- Hybrid zoomed-in simulation: Veltmaat et al.
- We would like to compare the wave and fluid formulation

Numerical Methods

- We build two methods to simulate the FDM

- Schrodinger-Poisson solver: operator splitting + Runge Kutta

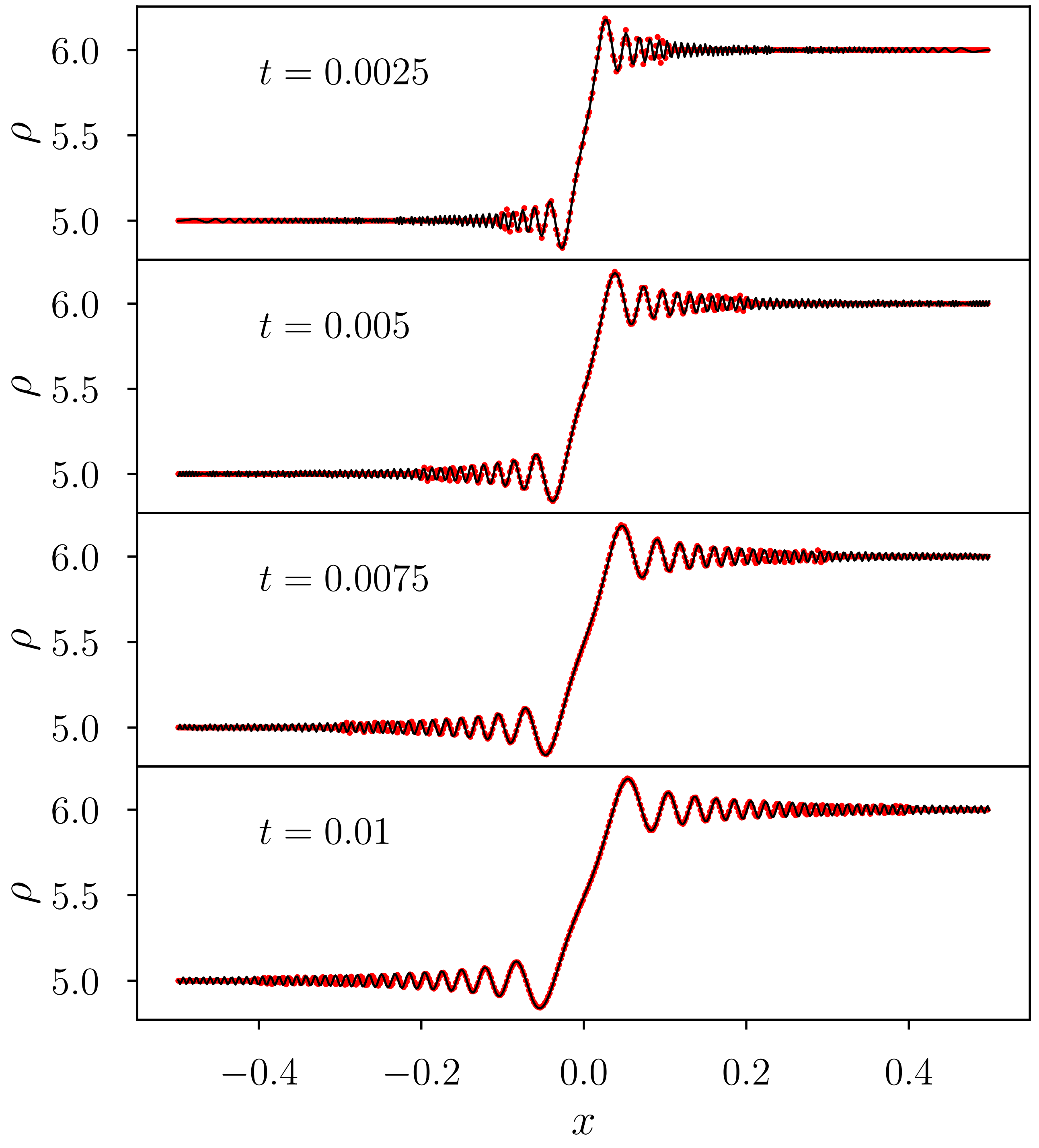
$$\begin{aligned}\tilde{\psi}(t+\Delta t) &= e^{(K+V)\Delta t}\tilde{\psi}(t) \\ &= e^{V\Delta t/2}e^{K\Delta t}e^{V\Delta t/2}\tilde{\psi}(t) + \mathcal{O}(\Delta t^3)\end{aligned}$$

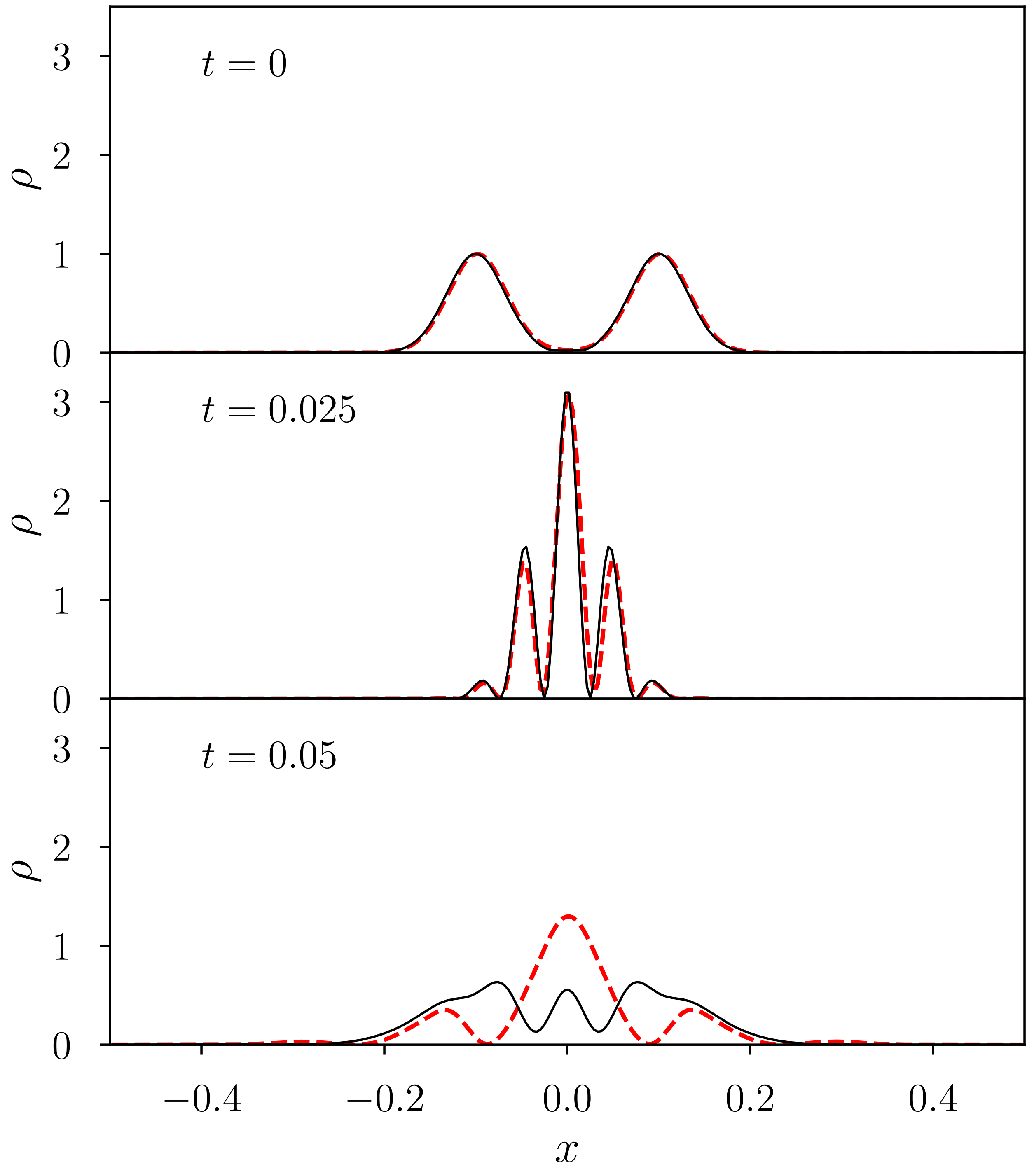
- Fluid solver: Zeus-3D (Stone & Norman)

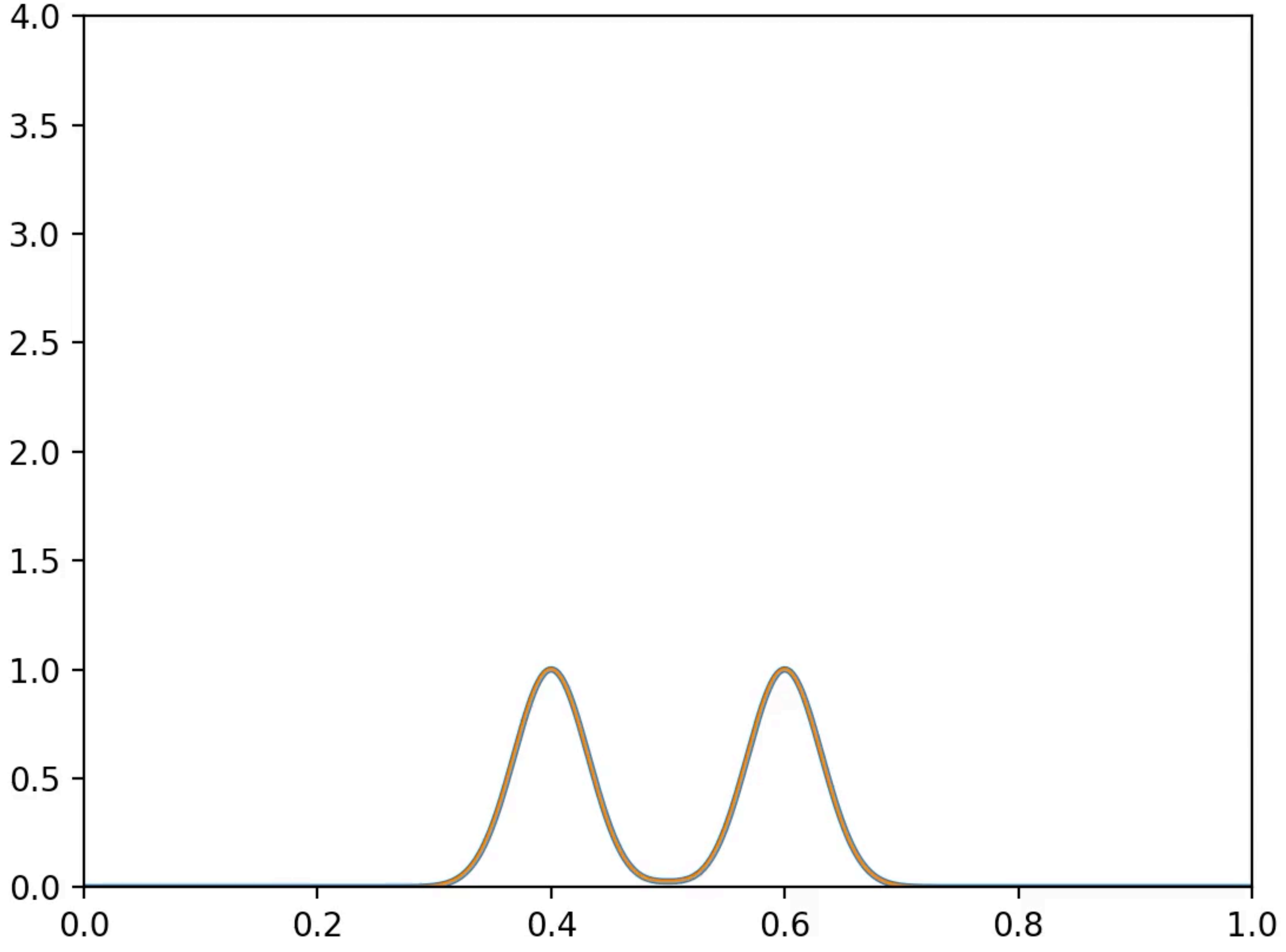
- The two solvers are built as module in the ENZO code and utilize the existing Poisson solver

Fluid Code

- Fluid code fails at the destructive interference where density becomes zero. Velocity and quantum pressure is actually infinite!






4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Why fluid solver fails? More technical points

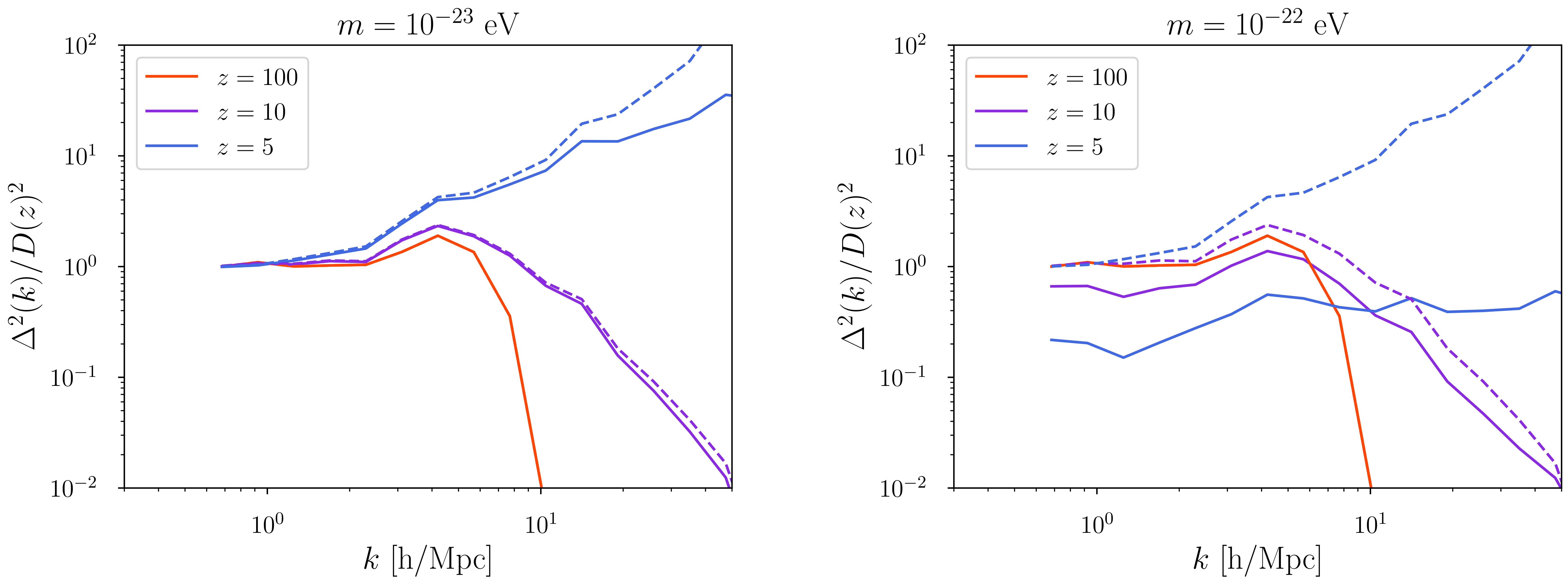
1. Phase and velocity are not well-defined at zero density.

2. $p \sim \nabla^2 \log \rho$ At zero density, any truncation error on ρ induces $\mathcal{O}(1)$ error on p.

3. More fundamentally, fluid solver is for **hyperbolic** system with **finite** characteristic signal speed. Schrodinger equation is intrinsically a **parabolic** system. The signal speed is **infinite**!

Wave Code

- No problem at the destructive interference
- Very demanding of resolution, need to resolve the de Broglie wavelength even to get the large scale right!



	Advantage	Disadvantage
Schrodinger-Poisson Solver	Correct dynamics of the interference pattern	Must resolve the de Broglie wavelength to get the correct large scale structure, computationally expensive
Fluid Solver	Correct large scale structure without resolving the de Broglie wavelength	Unable to follow the correct dynamics past the vanishing density

Application to Lyman-α Forest

- Previous study (Irisic et al. 2017, Armengaud et al. 2017) using XQ-100, HIRES/MIKE and SDSS data exclude FDM mass smaller than 10^{-22}-10^{-21} eV.
- They don’t include detailed physical modelling of Lyman-α forest.
- More importantly, they run N-body simulations with the FDM initial condition. Dynamical effects of FDM is not included!

Comparison Between FDM and CDM Dynamics

- Run FDM and CDM simulations with the same initial condition corresponding to FDM mass 10^{-22} eV.
- Compare the ratio of P_{CDM}/P_{FDM}.
- Gunn-Peterson approximation

$$\tau = A(1+\tilde{\delta})^2$$

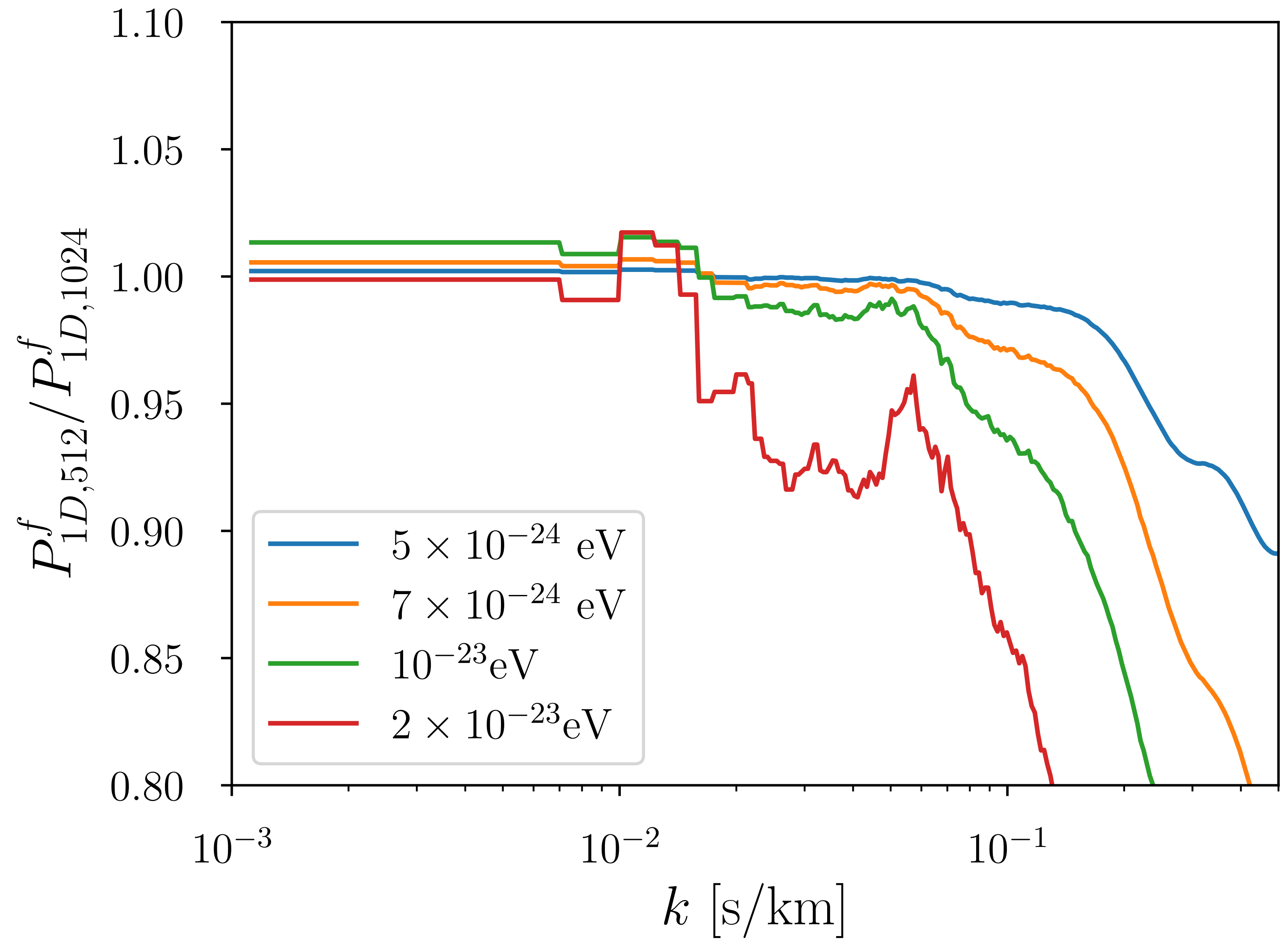
- Smoothed overdensity

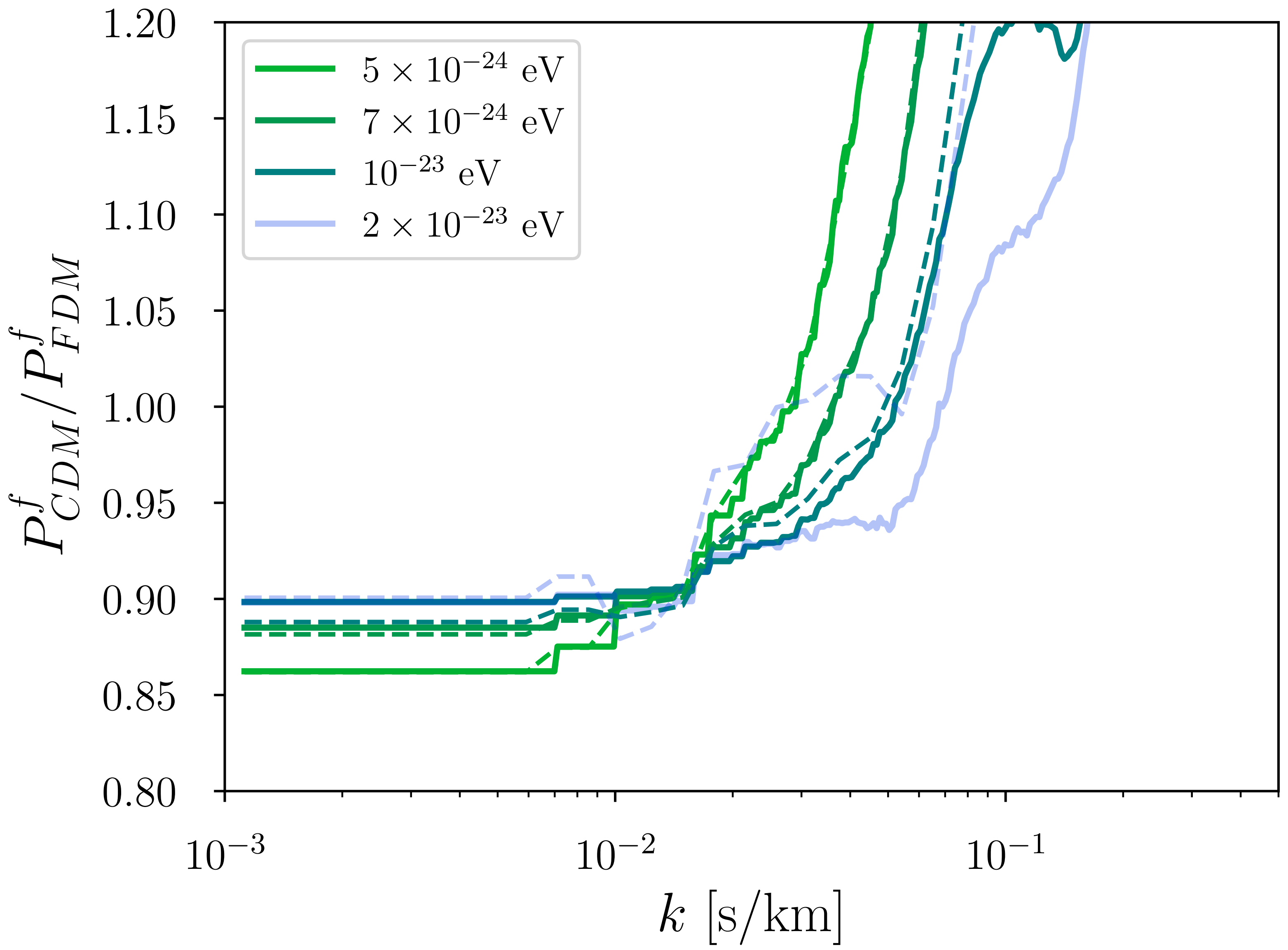
$$\tilde{\delta}(k) = \exp\left[-\left(\frac{k}{k_f}\right)^2\right]\delta(k)$$

- 1D flux spectrum

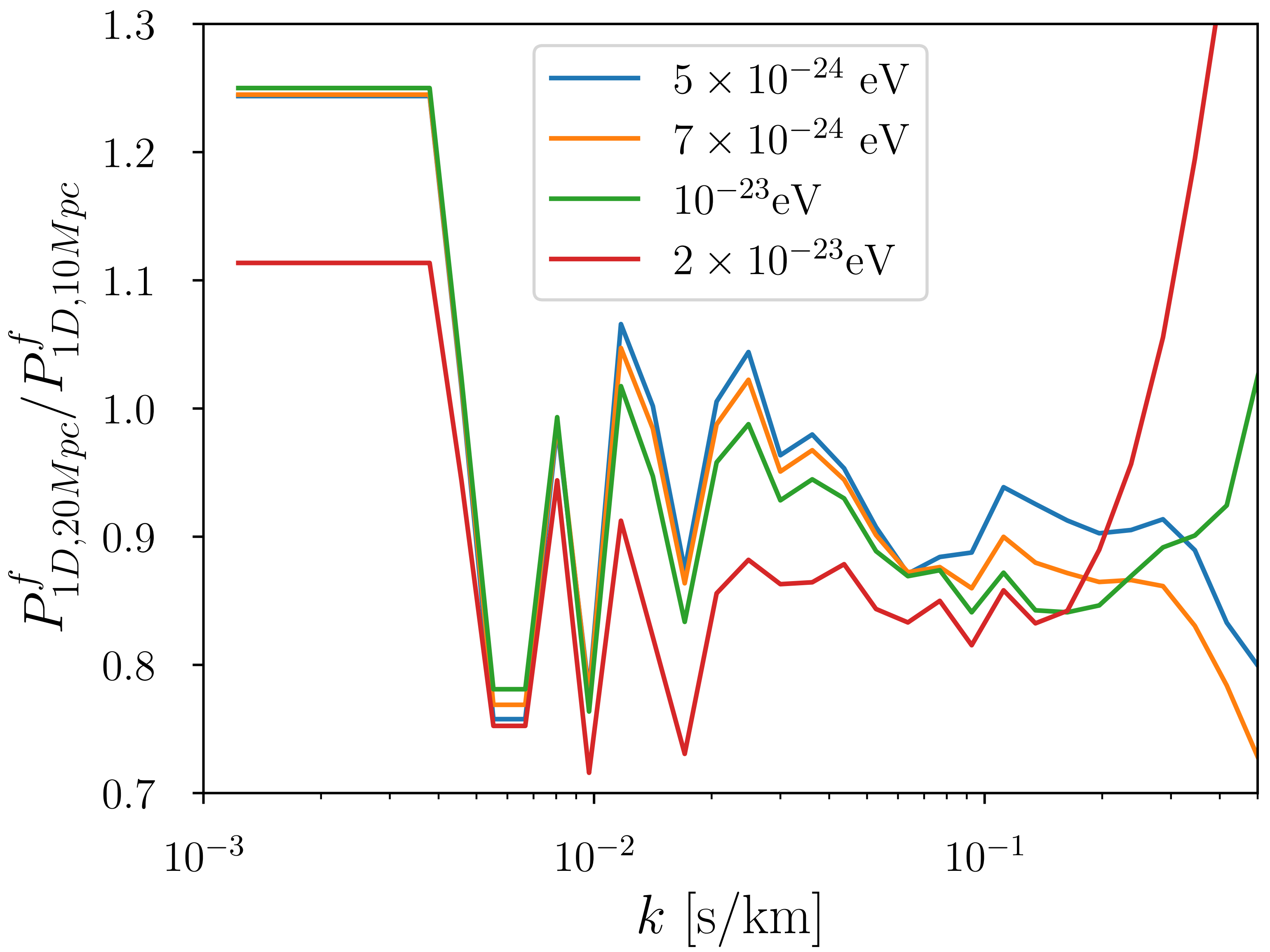
$$P^f(k) = \int_k^{\infty} \frac{k'\,\mathrm{d}k'}{2\pi} P^f_{3\mathrm{D}}(k')\,.$$

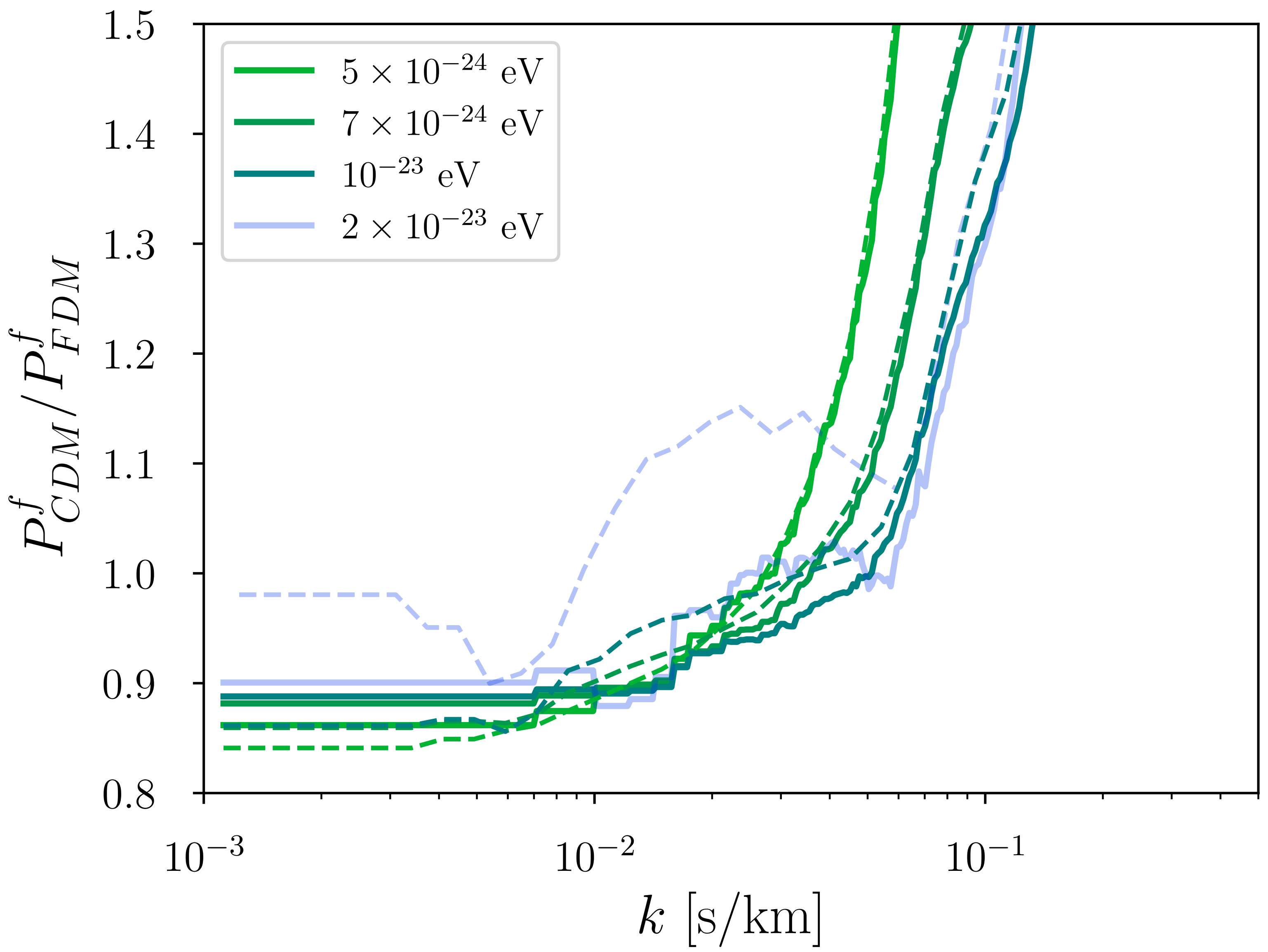
Convergence Test



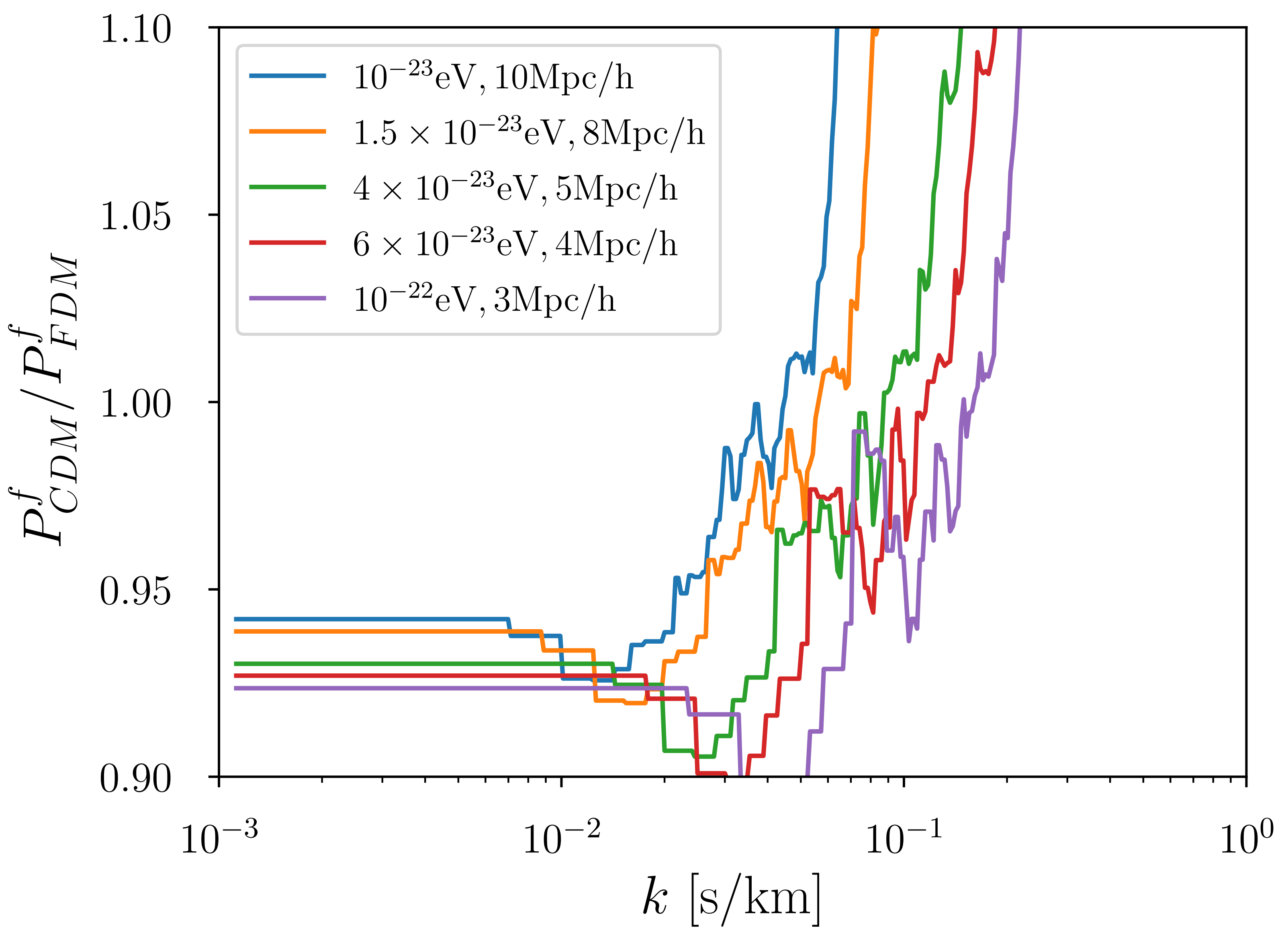


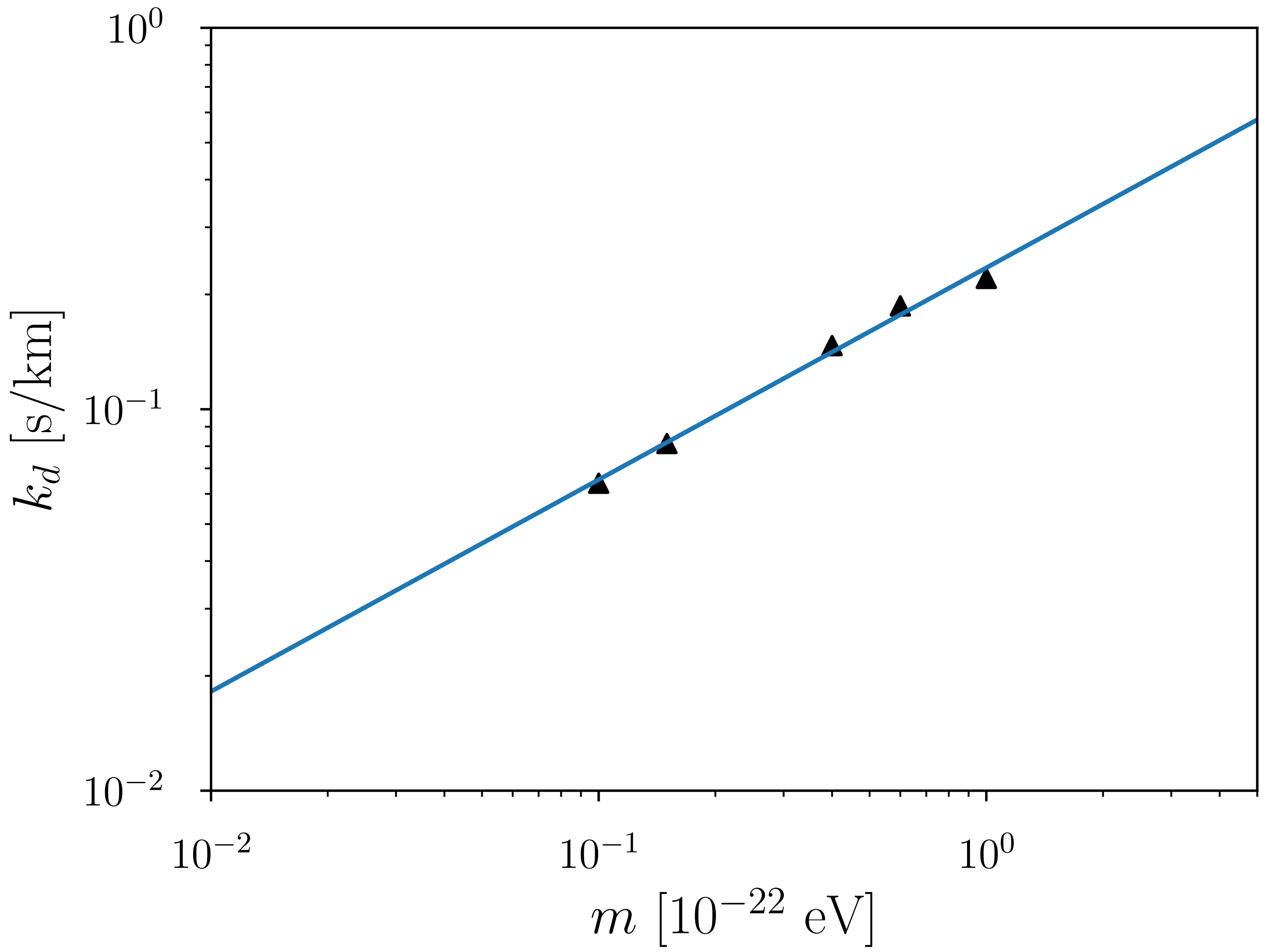
Box Size Effect





Consistent FDM mass in IC and Dynamics





$$k_d \approx 0.23\ \mathrm{s/km}\left(\frac{m}{10^{-22}\ \mathrm{eV}}\right)^{0.56}$$

Conclusion

- Fluid simulations can’t follow the correct dynamics where a destructive interference produces a zero density.

- Wave simulations is very demanding of resolutions. The de Broglie wavelength must be resolved.

- FDM and CDM dynamics agree on large scale flux spectrum (k<0.1s/km or >100kpc), but CDM dynamics has more power on small scales.

Analytical solution

- Taylor expansion of Ψ near the vortex on a surface

$$\Psi(\mathfrak{z}, \bar{\mathfrak{z}}) \simeq \mathfrak{z}\, \partial\Psi(0) + \bar{\mathfrak{z}}\, \bar{\partial}\Psi(0) + \cdots \simeq a\mathfrak{z} + b\bar{\mathfrak{z}} + \cdots$$

$$\mathfrak{z} \equiv x + iy\,, \qquad \bar{\mathfrak{z}} \equiv x - iy,$$

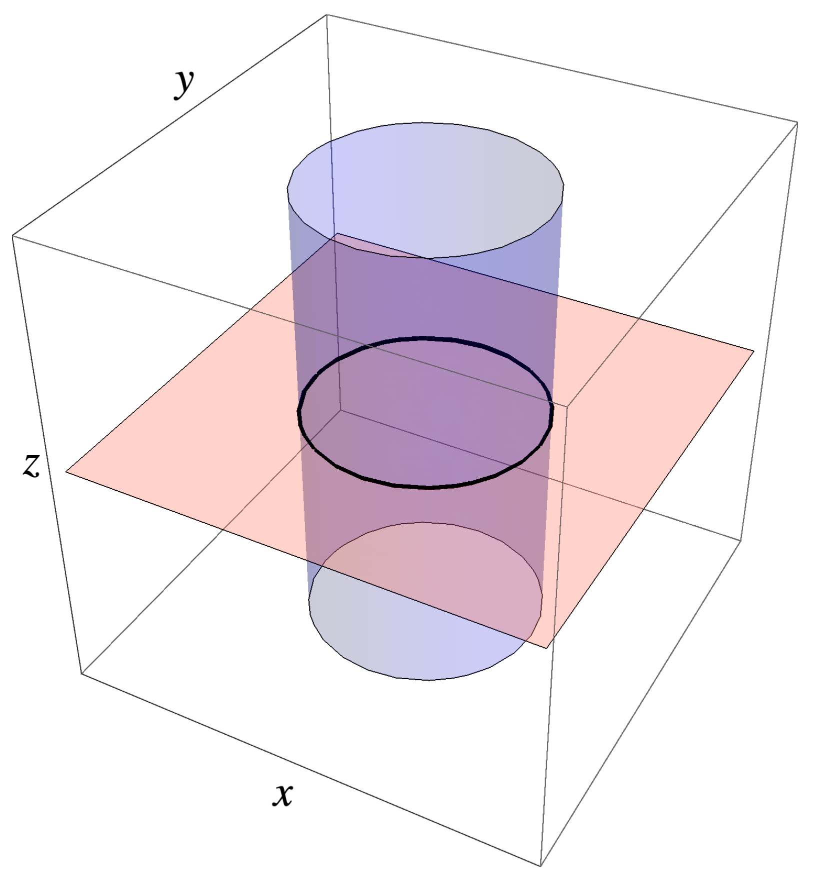
- The phase winds by multiples of $\pm 2\pi$ around the vortex.

- Static axis-symmetric solution $\quad \partial\bar{\partial}\Psi(\mathfrak{z}, \bar{\mathfrak{z}}) = 0.$

- Simplest case: $\Psi = z$ or $\bar{z}$. Density increase as $\rho^2 = x^2 + y^2$

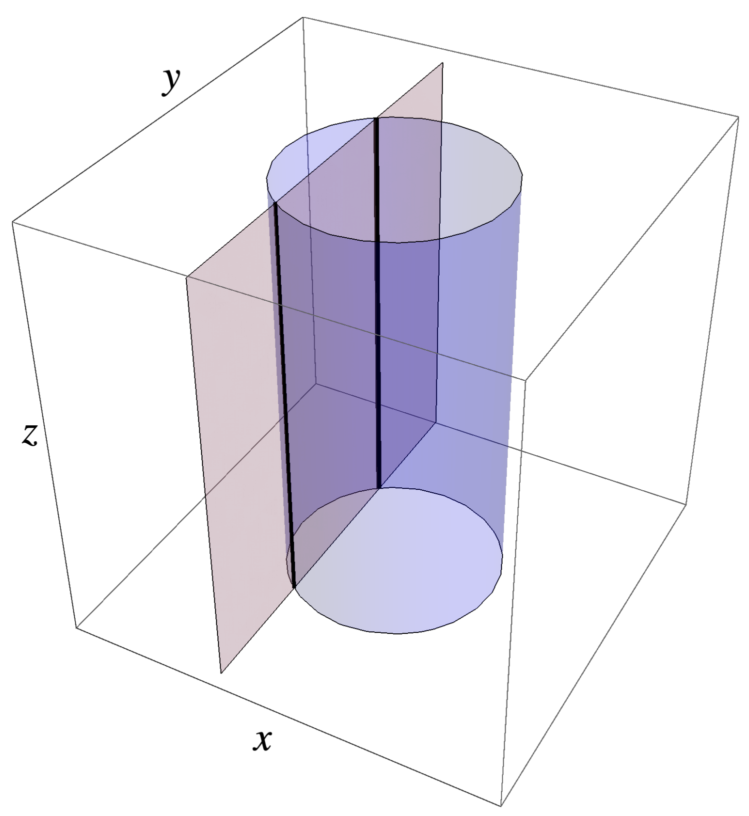
- Vortex ring

$$\Psi_{\mathrm{ring}}(\vec{x},t) = x^2 + y^2 - R^2 + i\left(-az + \frac{2t}{m}\right).$$



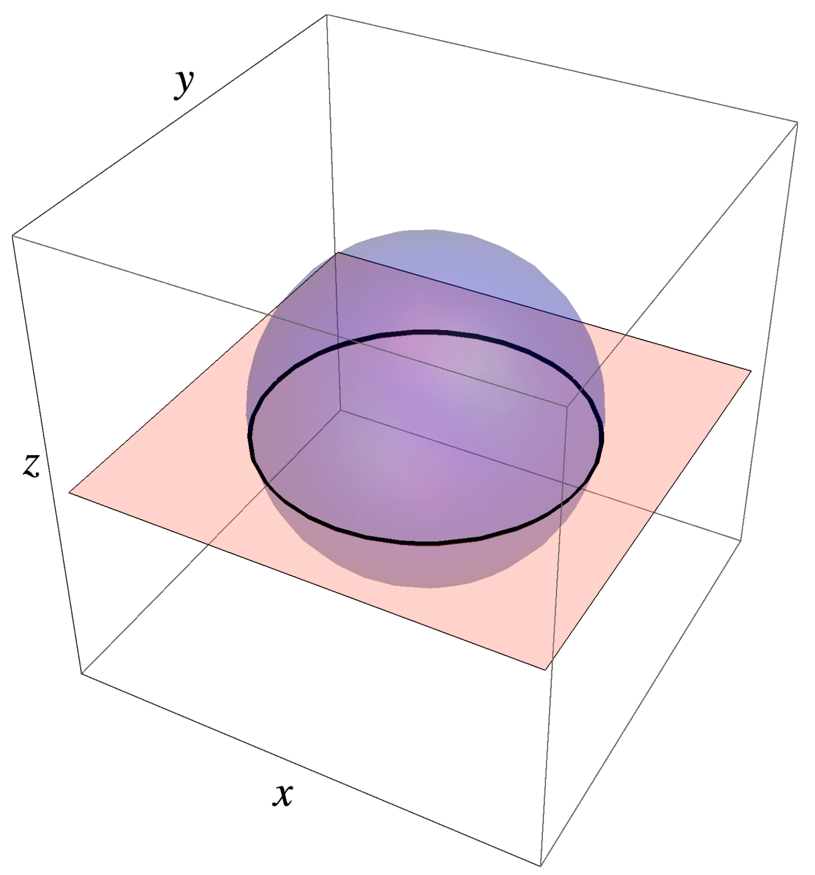
- Nucleation

$$\Psi_{\mathrm{v\bar{v}}}(\vec{x},t) = x^2 + y^2 - R^2 + 2i\left(-Rx + \frac{t}{m}\right).$$



- Nucleation

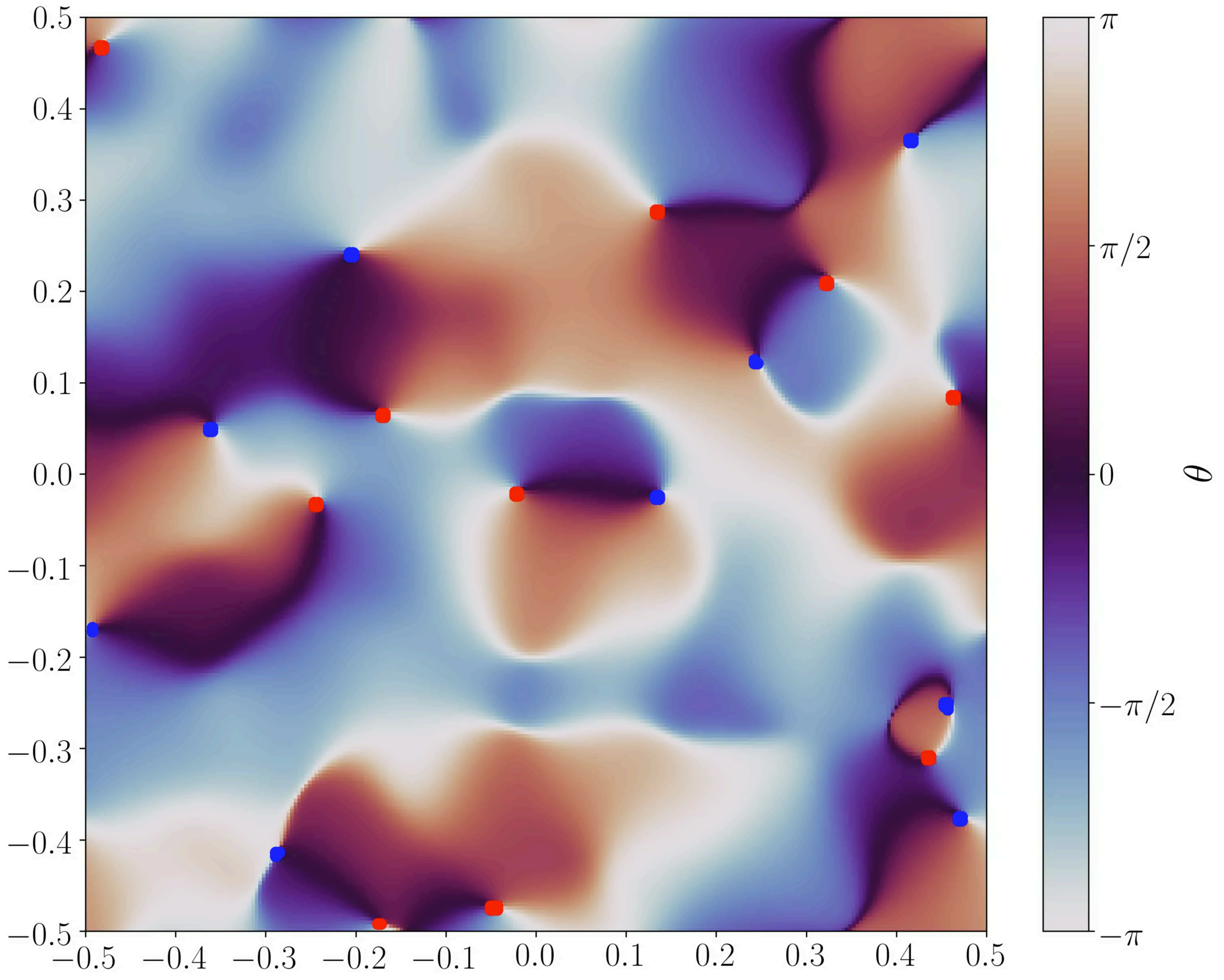
$$\Psi_R(\vec{x},t) = \rho^2 + z^2 - R^2 + i\left(-2Rz + \frac{3t}{m}\right),$$



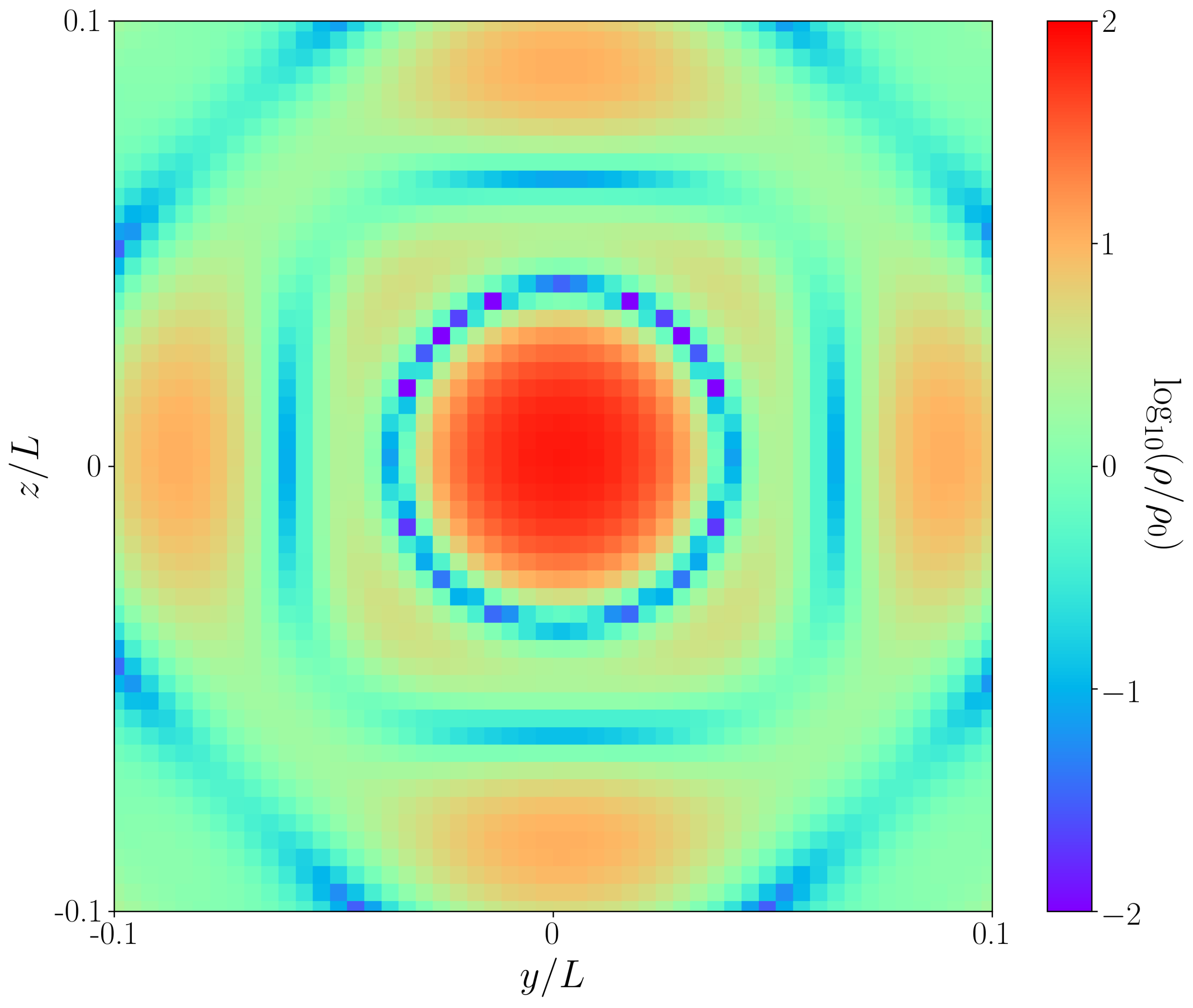
Numerical Realizations — 2D

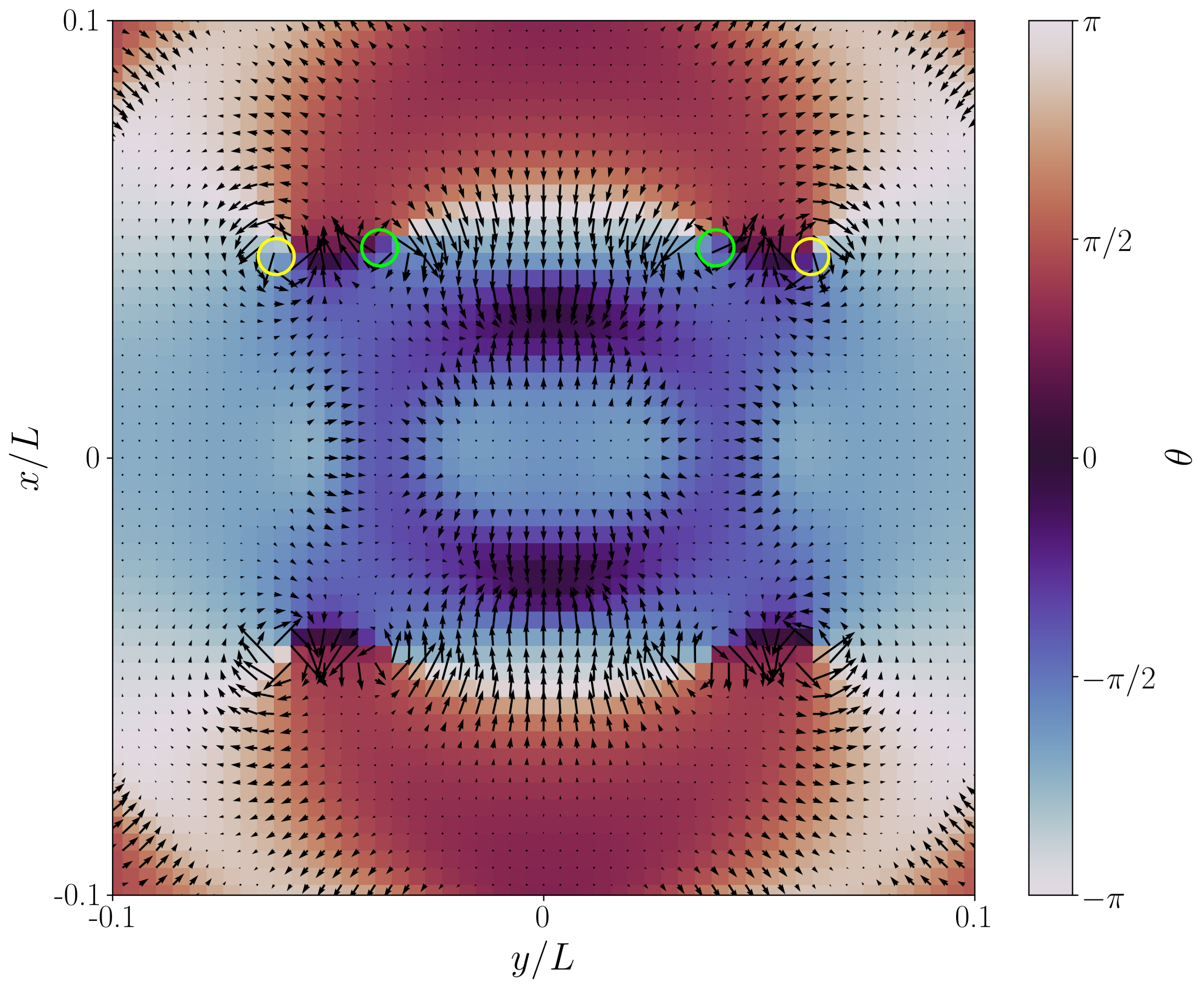
- Initial condition: Real and Imaginary are independent Gaussian with spectrum e^{-k^2/k_{max}^2}

- We follow the dynamics of the Schrodinger equation.



Vortex profiles from simulations

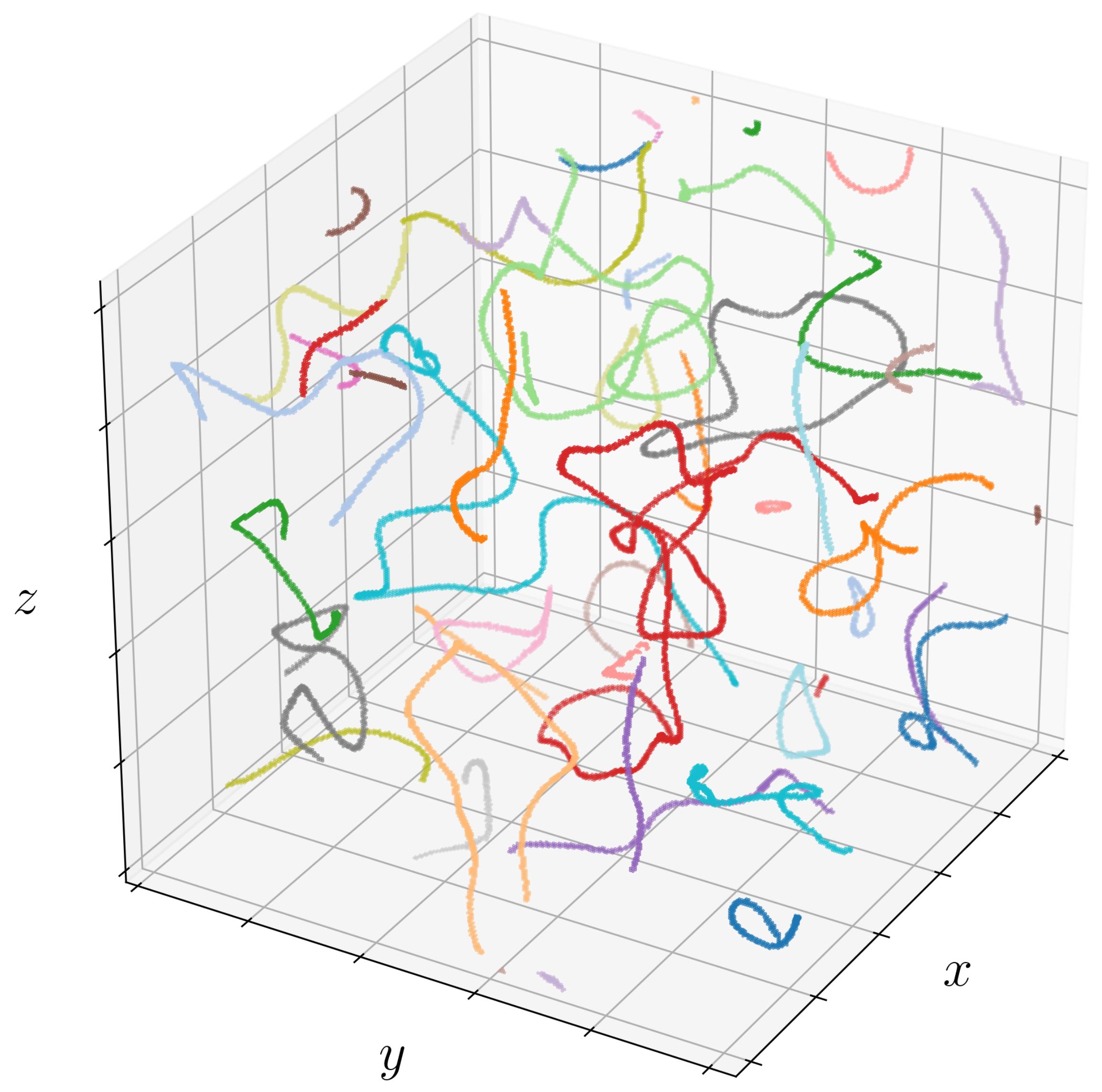


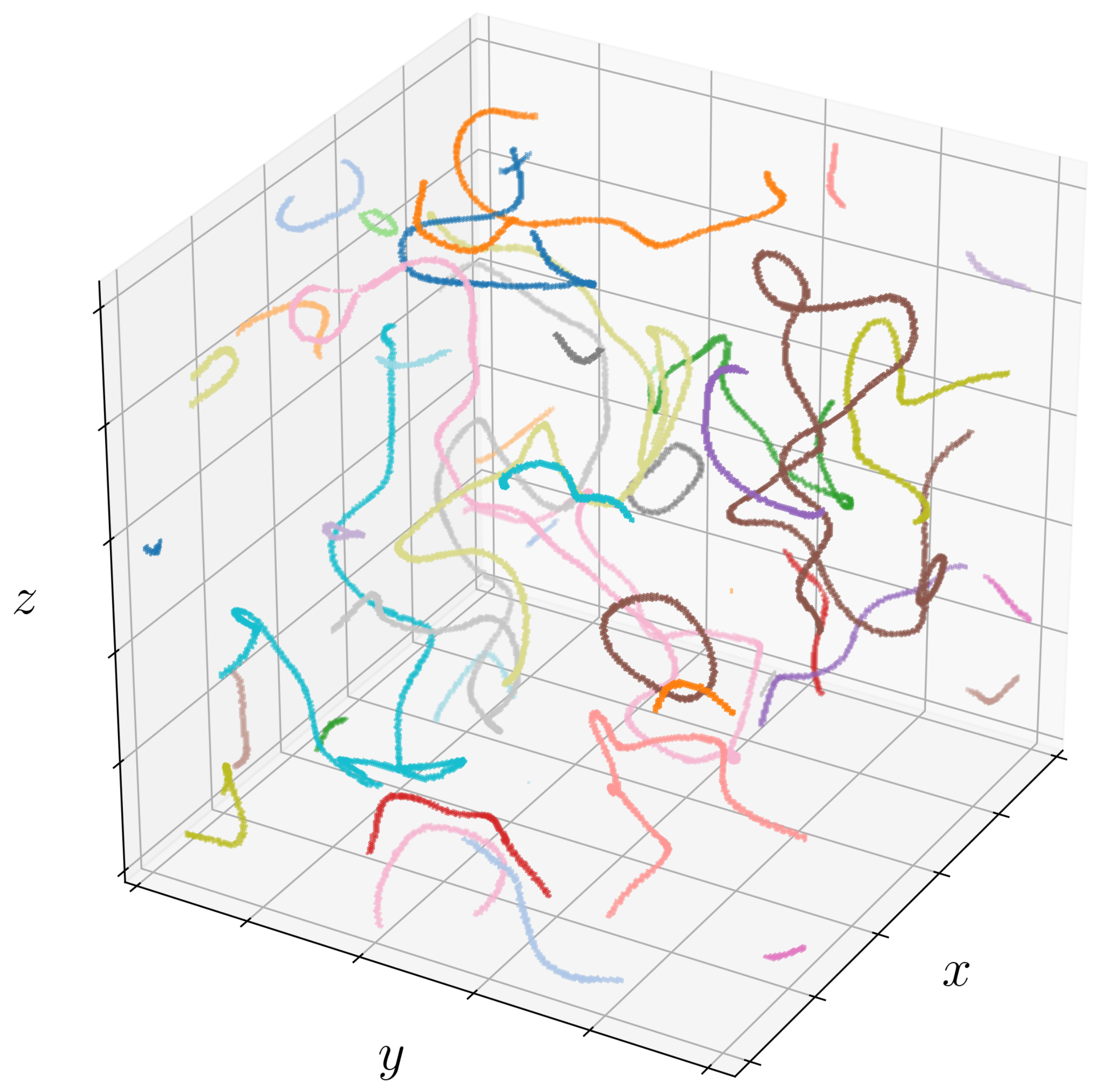


- Density slice
- Phase and velocity vector

Numerical Realizations — 3D

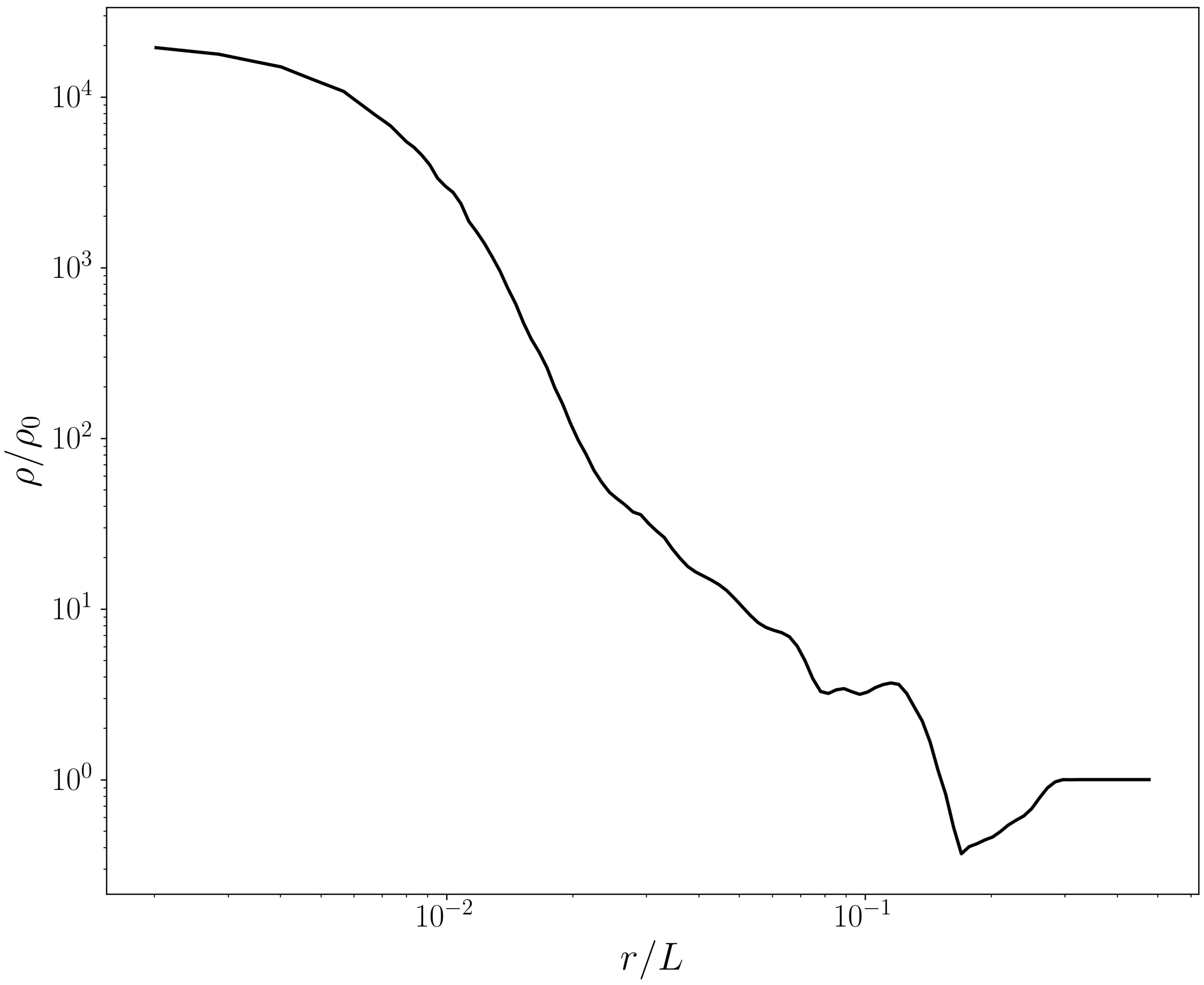
- Random field, no gravity





- Real and Imaginary are independent Gaussian with spectrum e^{-k^2/k_{max}^2}

- $\tilde{\psi}(k) = e^{-k^2/k_{max}^2} e^{i\beta}$, β is a uniformly random variable in $[0,2\pi)$


10^4
10^3
10^2
10^1
10^0
ρ/ρ_0
10^{-2}
10^{-1}
r/L

Distribution of vortex lines

- Vortex rings can form an initial configuration with no net angular momentum.
- The typical size of vortices is found to be the de Broglie wavelength.
- The density of vortex lines is roughly 1 per de Broglie wavelength.

Observational Signature

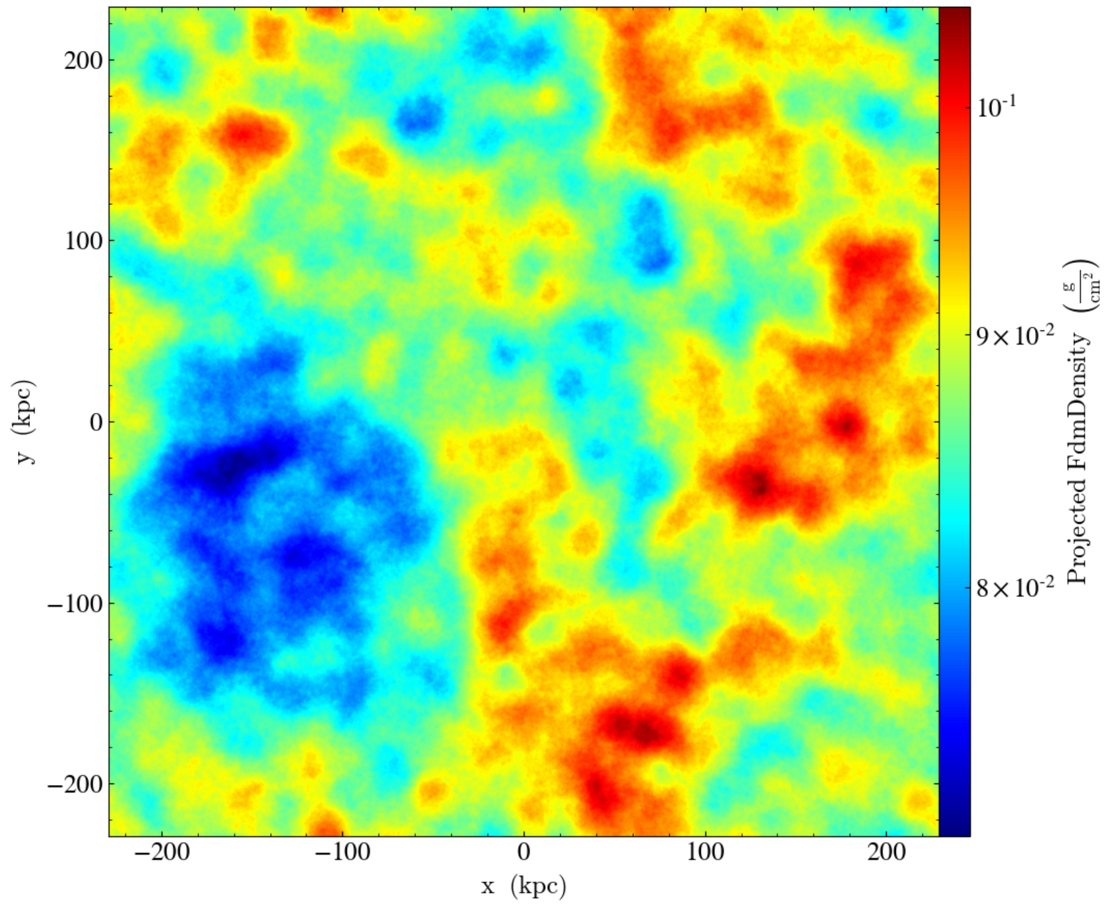
- Micro(de)lensing
- Variation of pulsar timing, Shapiro delay of pulses
- Dynamical effects, heating of stellar streams

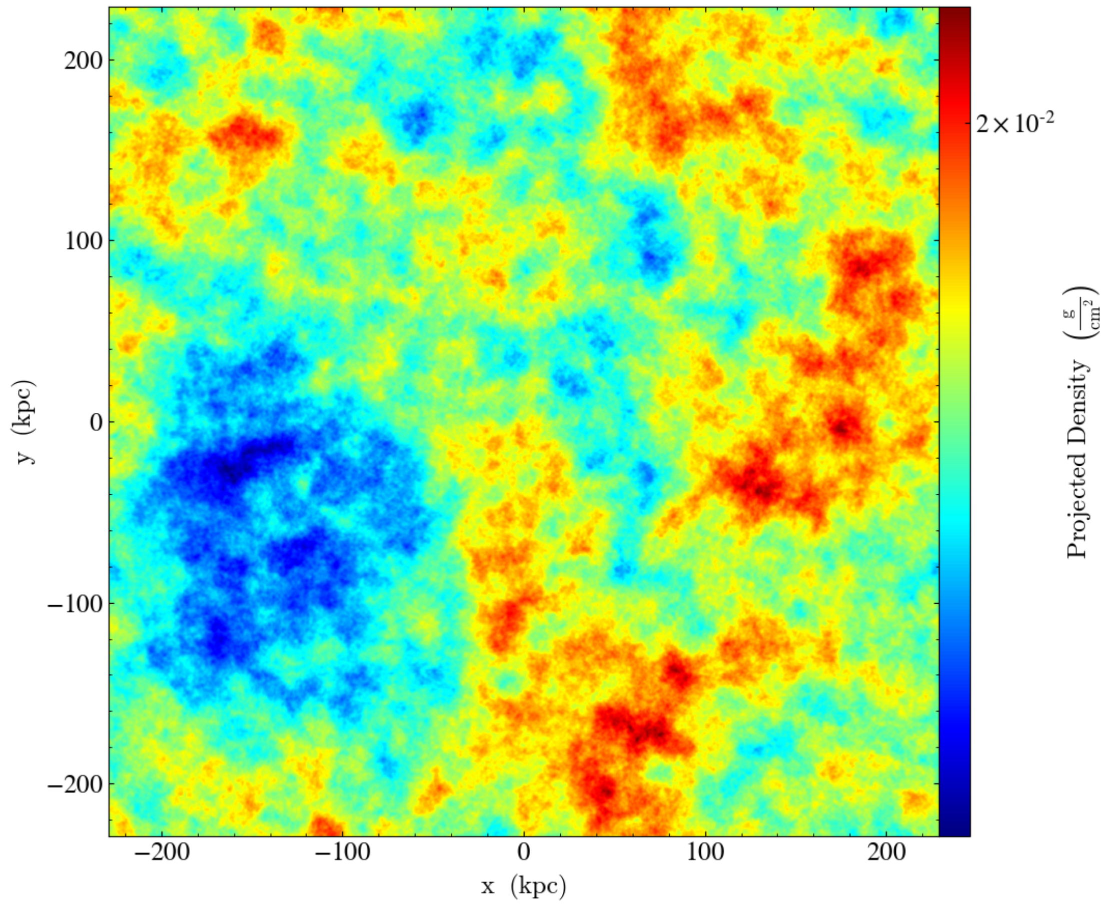
Future Work

- FDM + baryons or FDM + Nbody: My code can do simulations of any combination of species with chemistry and feedback.

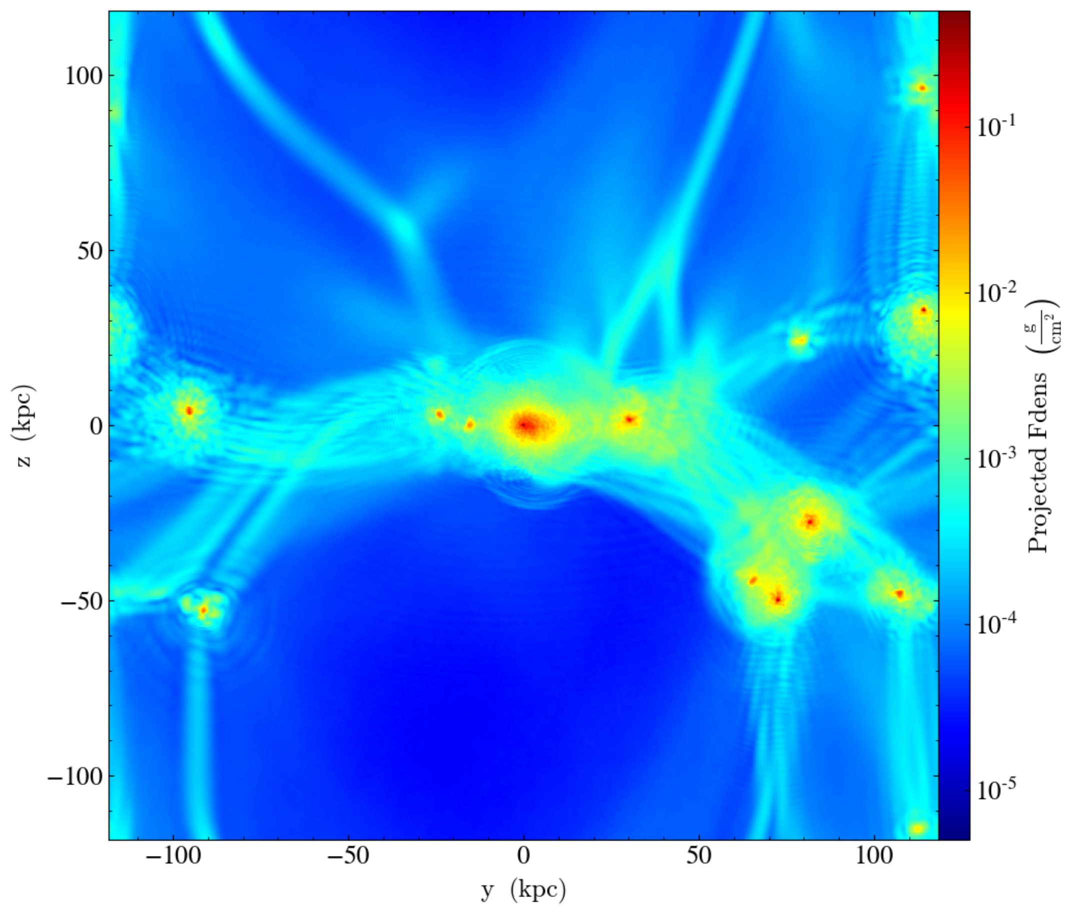
- The aim to constrain FDM model by detailed Lyman-α modelling, galactic morphology, probes of Epoch of Reionization, galaxy and star formation.

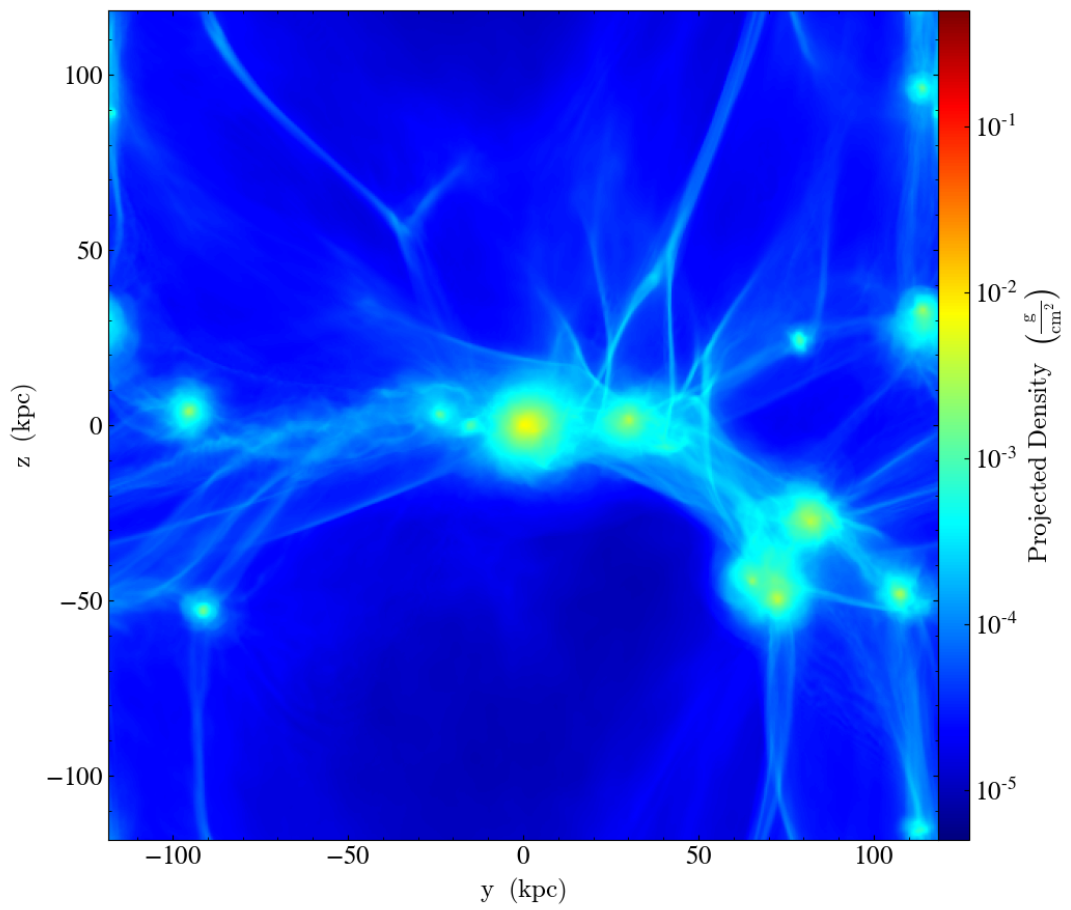
z=30





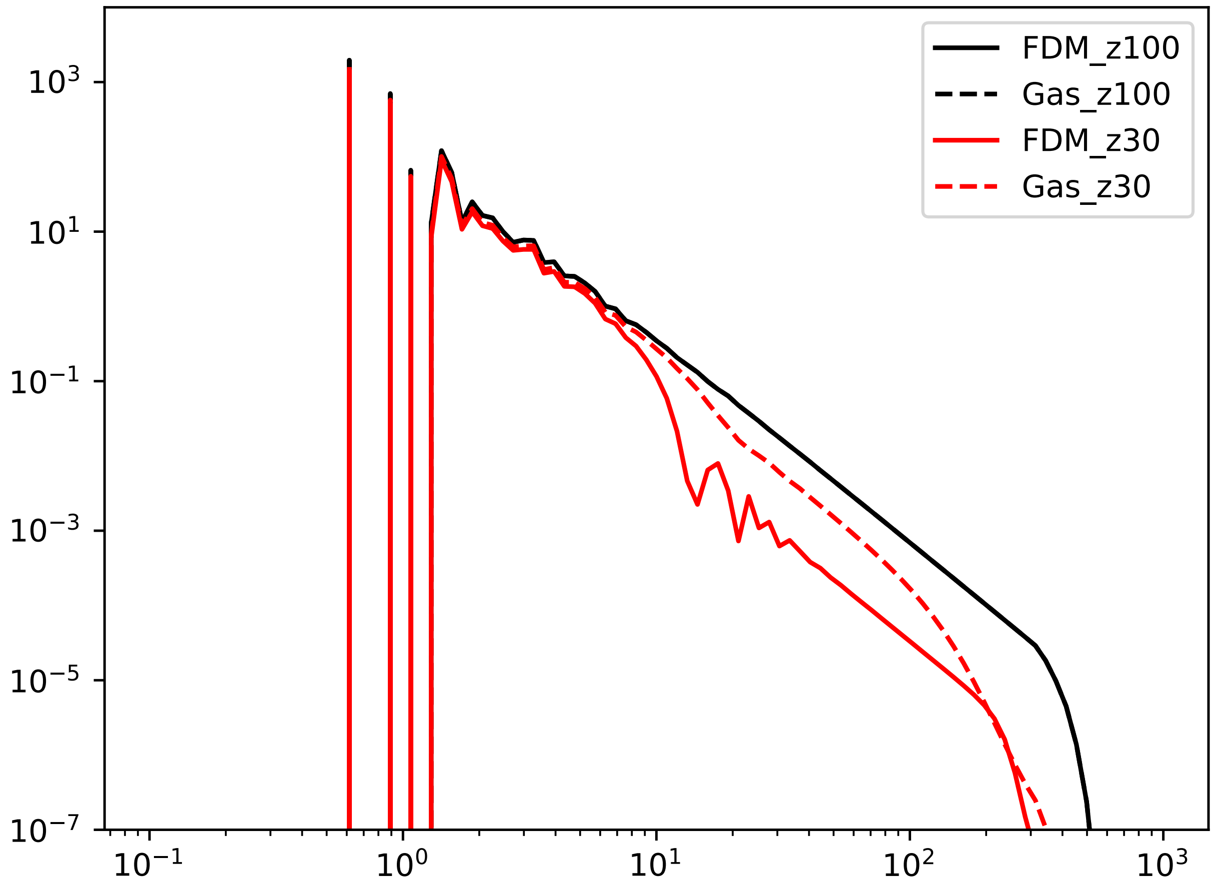
z=5





FDM

Baryons



Future Work

- Strong lensing signal from FDM halos. Work in progress with Liang Dai. The interference pattern can lead interesting lensing signals, e.g. multiple Einstein rings from a single source.

- The relaxation of FDM halos. Unlike CDM, the soliton core in the FDM halo is found to oscillate much longer than the dynamical timescale. What is the effect on star cluster heating?

- Effects of FDM on stellar streams?

Future Work

- Hybrid approach: N-body/Fluid simulation on coarse grid and Schrodinger-Poisson solver on a zoomed-in box to study the detailed structure of the DM halo.

- Most coding work has been DONE!!! If you are interested, please contact me.

- I am looking forward to collaborate with both theorists and observers.

Thank you for your attention!

Wave Perturbation Theory

- To first order

$$\delta = (\delta\psi + \delta\psi^*)/\bar{\psi}\,, \qquad v = \frac{1}{2ia\bar{m}\bar{\psi}}\boldsymbol{\nabla}(\delta\psi - \delta\psi^*)$$

$$\frac{\delta\psi - \delta\psi^*}{\bar{\psi}} \sim \frac{a\bar{m}v}{k} \sim \frac{a^2\bar{m}H}{k^2}\delta\,,$$

- The smallness of v requires $a^2\bar{m}H < k^2$

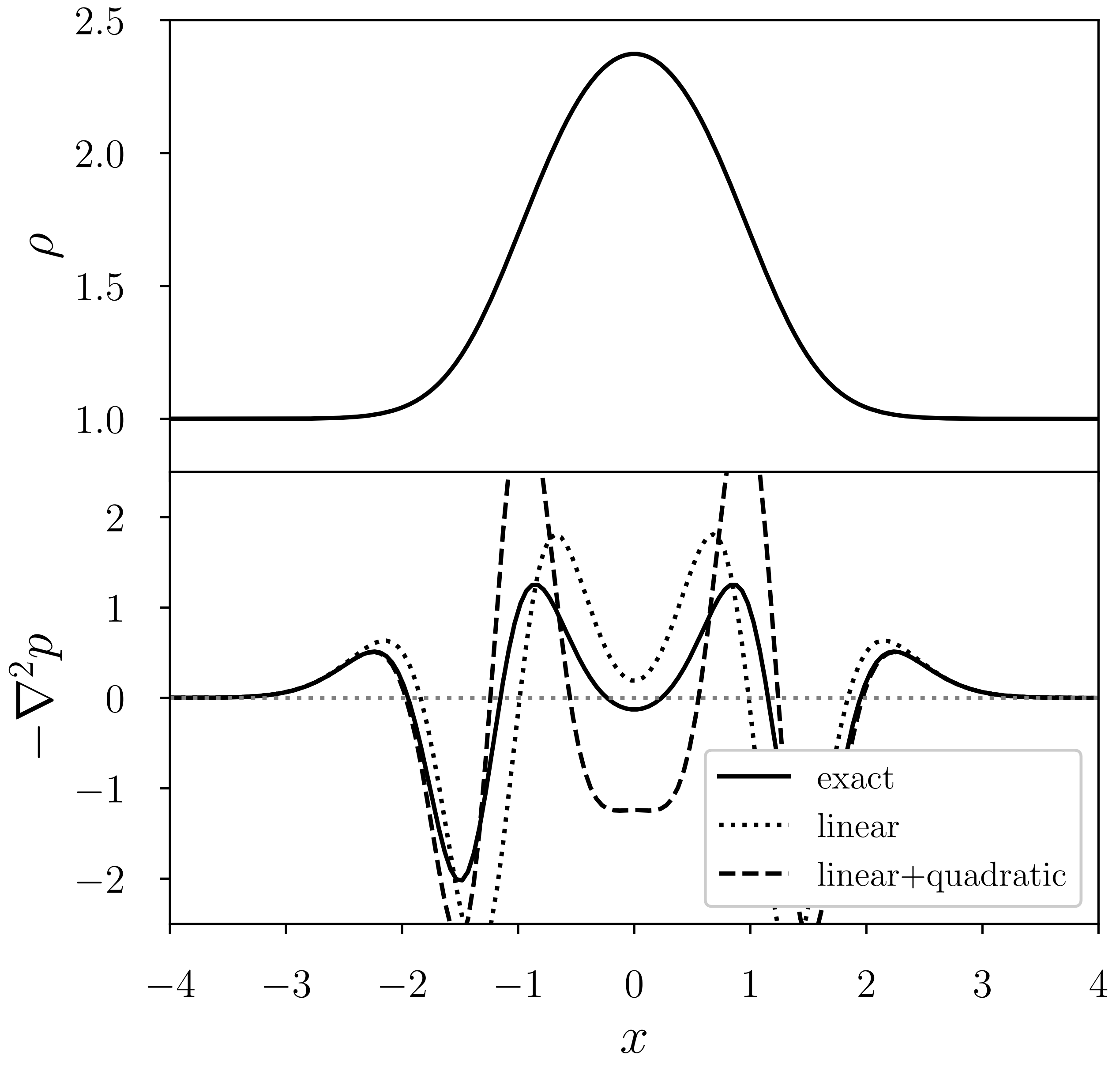
- The wave perturbation theory breaks down much earlier than fluid perturbation theory.

Fluid Perturbation Theory

- $$\partial_\eta \Theta + \frac{\partial_\eta a}{a}\Theta + \partial_i(v^j \nabla_j v^i) = -4\pi G\bar{\rho}a^2\delta$$
  $$+\frac{1}{4a^2\bar{m}^2}\nabla^2\left(\nabla^2\delta - \frac{1}{4}\nabla^2\delta^2 - \frac{1}{2}\delta\nabla^2\delta + ...\right)$$

- 1st order always opposes the gravity, but not necessarily correct!

$$\rho = 1 + \frac{1}{1 + 0.85\exp x^2}$$



1-Loop Matter Power Spectrum